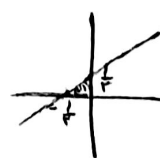


|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|
| $2y = 2x - 1 \xrightarrow{\text{دارد}} 2x = 2y - 1 \rightarrow y = \frac{2x+1}{2} \rightarrow f'(x) = \frac{2x+1}{2} = \frac{1}{2}x + \frac{1}{2}$ $\frac{1}{2}x + \frac{1}{2} = 0 \rightarrow x = -\frac{1}{2}$ $f'(-\frac{1}{2}) = \frac{1}{2}$  $\Rightarrow S = \frac{(Ay)(Ax)}{2} = \frac{(\frac{1}{2})(\frac{1}{2})}{2} = \frac{1}{8}$                                                                                                                                                                                                                                                      | ۱ |
| <p>الف) <math>y = x^2 - 2x + 2, x \in [1, +\infty)</math> راس ... <math>\frac{b}{2a} = 1 \Rightarrow</math> یک به یک است <math>\Rightarrow</math> <math>D_f^{-1} = [2, +\infty)</math></p> <p><math>y = (x-1)^2 + 2 \rightarrow \sqrt{y-2} + 1 = x \rightarrow f'(x) = y = 1 + \sqrt{2x-2} \Rightarrow D_f^{-1} = [2, +\infty)</math></p> <p>ب) <math>y = x^2 - 2x + 2, x \in [3, +\infty)</math> یک به یک است زیرا دامنه بعد از راس می باشد. <math>\Rightarrow</math> <math>D_f^{-1} = [4, +\infty)</math> است</p> <p>مرد تابع برای دامنه <math>[3, +\infty)</math> از ۴ به بعد از آن برابر با دامنه تابع معکوس است</p>                                                           | ۲ |
| $f(x) = 2x - \sqrt{4x^2 - 4x + 4} = 2x - \sqrt{(2x-1)^2} = 2x -  2x-1 $ <p><math>x=2 \Rightarrow y=3</math></p> <p><math>y = \begin{cases} 2x - 2x + 1 = 1 &amp; x \geq 1 \\ 2x - (2x-1) = 1 &amp; x &lt; 1 \end{cases}</math> شب معکوس است پس یک به یک نیست <math>\Rightarrow</math> <math>D_f^{-1} = (-\infty, 1)</math><br/> <math>D_f^{-1} = (-\infty, 2)</math><br/> <math>D_f^{-1} = (-\infty, 4)</math></p> <p><math>y = 2x - 1 \Rightarrow x = \frac{y+1}{2} \Rightarrow f^{-1}(y) = \frac{y+1}{2}</math></p> <p><math>S = S_{\square} + S_{\Delta} = 4 + \frac{4}{2} = 6</math></p>  | ۳ |
| $y = \frac{x}{\sqrt{x^2-5}} \rightarrow \frac{x^2}{x^2-5} = \frac{x^2}{x^2-5} \rightarrow x^2 = x^2 \Rightarrow x_1 = \pm x_2 \times \rightarrow x_1 = x_2$ <p>معکوس یک به یک است</p> <p><math>f^{-1}(x) = x = \frac{y}{\sqrt{y^2-5}} \rightarrow x^2 = \frac{y^2}{y^2-5} \rightarrow y^2 = \frac{5x^2}{x^2-1} \Rightarrow y = \pm \sqrt{\frac{5x^2}{x^2-1}}</math></p> <p>برای <math>y = \frac{\sqrt{5}x}{\sqrt{x^2-1}}</math> ضابطه تابع معکوس</p>                                                                                                                                                                                                                               | ۴ |
| <p>صعودی است <math>\Rightarrow (x-1)^2 - 4 + k, x \geq 1</math></p> <p>صعودی است <math>\Rightarrow -(x-2)^2 + 9 + 3k, x &lt; 2</math></p> <p><math>\rightarrow x=1 \Rightarrow 1+3k \quad x=2 \Rightarrow 1+3k \Rightarrow 1+3k &lt; -4+k \rightarrow k &lt; -5</math></p> <p><math>x=2 \Rightarrow k-4 \quad x \geq 2 \Rightarrow</math> یک به یک است</p>                                                                                                                                                                                                                                                                                                                         | ۵ |

$$f(x) = 3x + 1/x$$

$$\rightarrow \begin{cases} f(x) = 3x & x \neq 0 \\ f(x) = 1/x & x = 0 \end{cases} \Rightarrow \begin{cases} f^{-1}(x) = \frac{x}{3} \\ f^{-1}(x) = \frac{x}{3} \end{cases}$$

$$(1, 1) \in f \Rightarrow (1, 1) \in f^{-1} \quad (-1, -1) \in f \Rightarrow (-1, -1) \in f^{-1}$$

$$g(x) = ax + b/x \quad \begin{cases} fa + fb = 1 \\ -fa + fb = -1 \end{cases} \rightarrow \begin{cases} b = \frac{1}{x} \\ a = \frac{x}{x} \end{cases}$$

$$\rightarrow ab = \frac{x}{x} \cdot \left(\frac{1}{x}\right) = \frac{1}{x}$$

$$f(x) = \frac{3x+1}{x+1} \quad f(1) = \frac{10}{1+1} = 5 \rightarrow m = -3 \rightarrow f(x) = \frac{3x+1}{x-3}$$

$$f^{-1}(x) = \frac{3y+1}{y-3} = x \rightarrow y = \frac{-3x-1}{x-3} = \frac{3x+1}{3-x} \rightarrow f^{-1}(x) = f(x)$$

$$f \circ f^{-1}(x) = f \circ f(x) = \frac{3\left(\frac{3x+1}{3-x}\right)+1}{\left(\frac{3x+1}{3-x}\right)-3} = \frac{12x}{12} = x$$

$$\rightarrow y = f \circ f^{-1}(x) + f^{-1}(x) = x + \frac{3x+1}{3-x} = x \rightarrow \frac{3x+1}{3-x} = 0 \Rightarrow x = -\frac{1}{3}$$

$$3-2x = u \rightarrow x = \frac{3-u}{2} \rightarrow f(u) = 3 - g\left(\frac{3-u}{2}\right)$$

$$\rightarrow f(x) = 3 - g\left(\frac{3-x}{2}\right)$$

$$\rightarrow 3-y = g\left(\frac{3-x}{2}\right) \rightarrow g^{-1}(3-y) = \frac{3-x}{2} \rightarrow x = 3 - 2g^{-1}(3-y)$$

$$f^{-1}(y) = 3 - 2g^{-1}(3-y) \rightarrow f^{-1}(3) = 3 - 2g^{-1}(0)$$

$$\rightarrow 2g^{-1}(0) + f^{-1}(3) = 2g^{-1}(0) + 3 - 2g^{-1}(0) = 3$$

$$y = \frac{5x-12}{1-x} \rightarrow y = f(x) \rightarrow y = -f(-x) \rightarrow y = f(-x) = \frac{-5x-12}{1+x}$$

$$\rightarrow f(x) = \frac{3x+4}{x-1} \rightarrow x = \frac{y+4}{y-3}$$

$$\rightarrow f^{-1}(x) = \frac{2+x}{2-x}$$

$$\rightarrow f(x) = \frac{3x+4}{x-1} \rightarrow \frac{3x+4}{x-1} = \frac{2+x}{2-x}$$

$$\rightarrow x^2 - 2x - 4 = 0 \rightarrow x = \frac{2 \pm \sqrt{4+16}}{2} = \frac{2 \pm \sqrt{20}}{2} = 1 \pm \sqrt{5}$$

$$y = f(x) = x + f[x] \quad f(y) = y + f[y] = x \rightarrow y = x - f[y]$$

$$[y + f[y]] = [x] \rightarrow \omega[y] = [x] \Rightarrow y = x - f[y] = x - \frac{f[x]}{\omega} = g(x)$$

$$\rightarrow [y] = \frac{[x]}{\omega}$$

$$\rightarrow f(x) - g(x) = x + f[x] - x - \frac{f[x]}{\omega} = \frac{1-\omega}{\omega} [x]$$