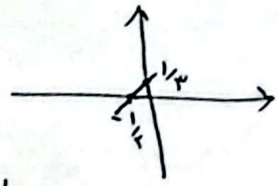
متن تشریحی

۱-

$2y = 3x - 1$

$y = \frac{3x+1}{3}$ → $x=0, \; y = \frac{1}{3}$

→ $\frac{3x+1}{3} = 0 \quad 3x+1 = 0 \quad x = -\frac{1}{3}$



$$\boxed{S_\Delta = \frac{1}{2}\cdot\left(\frac{1}{3}\cdot\frac{1}{3}\right) = \frac{1}{18}}$$

۲-

الف) $y = x^2 - 2x + 3 \;,\; x \in [1, +\infty)$ $\quad y = (x-1)^2 + 2 \quad f^{-1}(x) = \sqrt{x-2} + 1$

$D_{f^{-1}} = [2, +\infty)$

ب) $y = x^2 - 2x + 3 \quad [3, +\infty)$

$f^{-1}(x) = \sqrt{x-2} + 1 \quad D_{f^{-1}} = [6, +\infty)$

$R_f = [6, +\infty)$

$\sqrt{x-2} + 1 = 3 \quad \sqrt{x-2} = 2$

$x - 2 = 4 \quad x = 6$

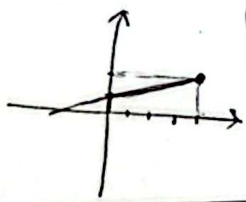
۳-

$f(x) = 2x - \sqrt{16 - 4x(4-x)} \quad 2x - \sqrt{16 - 16x + 4x^2} \quad 2x - \sqrt{4(2-x)^2} = 2x - 2(2-x) = 4x - 4$

$D_f = (-\infty, 2]$

$f(x) = 4x - 4 \quad f^{-1}(x) = \frac{x}{4} + 1$

$R_{f^{-1}} = (-\infty, 2]$



$$\boxed{S = \frac{(1+2)\times \cancel{4}}{\cancel{2}} = 6}$$

۴-

$y = \frac{x}{\sqrt{x^2-5}} \quad \frac{x_1}{\sqrt{x_1^2-5}} = \frac{x_2}{\sqrt{x_2^2-5}} \quad \frac{x_1^2}{x_1^2-5} = \frac{x_2^2}{x_2^2-5}$

$\cancel{x_2^2 x_1^2} - 5x_1^2 = \cancel{x_2^2 x_1^2} - 5x_2^2$

$x_1^2 = x_2^2 \quad x_1 = \pm x_2$

یک به یک نیست

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$$g(x)\begin{cases} x^2-4x+k, & x\geq 2 \quad \text{[illegible]} \quad k-4 \\ -x^2+6x+3k, & x<2 \quad \text{[illegible]} \quad 8+3k \end{cases}$$

$\cancel{8+3k < k-4}$

$3k < -12$

$\cancel{k<-6}$

$0<k$

$3k < 4-k \qquad 0<k<1$

$4k<4$

$k<1$

-۶

$f(x)=3x+|x|$

$$f(x)\begin{cases} 4x & x\geq 0 \\ 2x & x<0 \end{cases} \qquad g(x)\begin{cases} \frac{x}{4} & x\geq 0 \\ \frac{x}{2} & x<0 \end{cases}$$

$g(x)=\frac{3}{8}x-\frac{1}{8}|x|$

$a=\frac{3}{8}$

$b=-\frac{1}{8}$

$$\boxed{ab=\frac{3}{8}\times -\frac{1}{8}=-\frac{3}{64}}$$

-۷

$f(x)=\frac{3x+4}{x+m} \xrightarrow{x=2} \frac{6+4}{2+m}=-10 \qquad 2+m=-1$

$m=-3$

$f(x)=\frac{3x+4}{x-3}$

$f^{-1}(x)\rightarrow y=\frac{3x+4}{x-3}$

$yx-3y=3x+4 \qquad y=\frac{3x+4}{x-3} \qquad f^{-1}(x)=\frac{3x+4}{x-3}$

$yx-3x=3y+4$

$x(y-3)$ -

$f\circ f^{-1}=x$

$\cancel{f\circ f^{-1}}(x)+f^{-1}(x)=x$

$x+\frac{3x+4}{x-3}=x \qquad \frac{3x+4}{x-3}=0 \qquad 3x+4=0$

$3x=-4 \qquad \boxed{x=-\frac{4}{3}}$

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$f(۳-۲x)=۲-g(\frac{x}{۲})$ $\frac{x}{۲}=t'$ $x=۲t'$

$۳-۲x=t$ $f(t)=۲-g(\frac{۳-t}{۴})$ $g(t')=۲-f(۳-۴t')$

$x=\frac{۳-t}{۲}$

$۲-g(\frac{۳-t}{۴})=-۲$ $۲-f(۳-۴t')=۴$ $f(۳-۴t')=-۲$

$g(\frac{۳-t}{۴})=۴$ $g^{-1}(۴)=\frac{۳-t}{۴}$

$g(\frac{۳-t}{۴})=۲-f(۳-\cancel{۴}(\frac{۳-t}{\cancel{۴}}))=۴$ $۲-f(t)=۴$ $f(t)=-۲$

$-f(t)=۲$ $f^{-1}(-۲)=t$

$\boxed{۴(\frac{۳-t}{۴})+t=۳}$

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~~$\frac{۵x-۱۳}{x-۱}$ ۲ واحد به راست $\frac{۵x-۱۹}{x-۳}$ ۳ واحد به پایین $\frac{۲x-۱۴}{x-۳}$ $f(x)=\frac{۲x-۱۴}{x-۳}$~~

$y=\frac{۵x-۱۳}{۱-x} \xrightarrow{\text{قرینه}} \frac{۵x-۱۳}{x-۱}$ $\frac{۵(x-۲)-۱۳}{(x-۲)-۱}-۳=\frac{۵x-۲۳}{x-۳}-۳$

$\frac{۵x-۲۳-۳(x-۳)}{x-۳}=\frac{۵x-۲۳-۳x+۹}{x-۳}=\frac{۲x-۱۴}{x-۳}$ $f(x)=\frac{۲x-۱۴}{x-۳}$

$f^{-1}(x)=\frac{۳x-۱۴}{x-۲}$ $\frac{۳x-۱۴}{x-۲}=\frac{۲x-۱۴}{x-۳}$ $(x-۳)(۳x-۱۴)=(۲x-۱۴)(x-۲)$

$۳x^۲-۲۳x+۴۲=۲x^۲-۱۸x+۲۸$

$x^۲-۵x+۱۴=۰$ $x^۲-۵x+۱۴$

$(x-۷)(x+۲)=۰$ $x=-۲$ $x=۷$ $\boxed{x_۱+x_۲=-۵}$

$f(x)=x+۴[x]$

$[y]=[x]+۴[x]$ $\cancel{[x]=۵[x]}$ $f^{-1}(x)=[x]$

$\cancel{[x]=\frac{[y]}{۵}}$