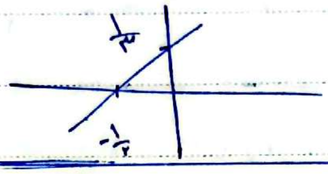
پارسا ملکی   دوازدهم ر A   تکلیف شماره ۱

$f(x) = 3y$, $f(x) = 3x - 1 \Longrightarrow f^{-1} : 3x + 1 = 3y \Longrightarrow f^{-1} = \frac{2x+1}{3} - 1$

$= \frac{2}{3}x + \frac{1}{3} \Longrightarrow$  $\frac{\frac{1}{2} + \frac{1}{3}}{2} = \frac{1}{12}$

۲- مورد الف) $y = x^2 - 2x + 3$, $x \in [1, +\infty)$ $-\frac{b}{2a} = 1 \longrightarrow$

$\Longrightarrow f^{-1} : x = (y-1)^2 + 2 \Longrightarrow y - 1 = \sqrt{x-2} \Longrightarrow y = \sqrt{x-2} + 1$

$D_{f^{-1}} \in [2, +\infty)$

مورد ب) $y = x^2 - 2x + 3$, $x \in [2, +\infty)$ $-\frac{b}{2a} = 1 \longrightarrow$

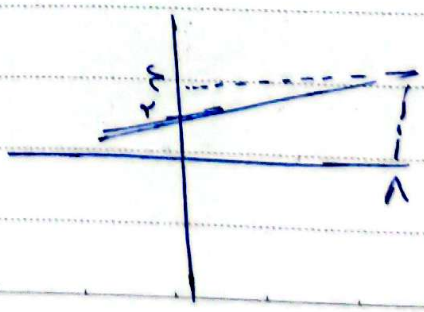
$y^{-1} = \sqrt{x-2} + 1$, $x \geq 3$, $R_{f^{-1}} \in [3, \infty)$

۳- $f(x) = 4x - \sqrt{16 - 8x(4-x)} = 4x - \sqrt{4(4-x)^2} = 4x - 2|4-x|$

مورد ب ⟹ اکیدا صعودی $\Longrightarrow$ if $D_f \in (-\infty, 4] \Longrightarrow$

$x \in (-\infty, 4] \Longrightarrow f(x) = 6x - 8 \Longrightarrow f^{-1}(x) = \frac{1}{6}x + 2$

$\Longrightarrow R_{f(x)} = (-\infty, 8] = D_{f^{-1}(x)}$



$\frac{4 \times 8}{2} = 24$

$y = \frac{x}{\sqrt{x^2-5}} \rightarrow \frac{x_1}{\sqrt{x_1^2-5}} = \frac{x_2}{\sqrt{x_2^2-5}} \Rightarrow \frac{x_1^2}{x_1^2-5} = \frac{x_2^2}{x_2^2-5}$ ۴-

$\Rightarrow \cancel{x_1^2x_2^2} - 5x_1^2 = \cancel{x_1^2x_2^2} - 5x_2^2 \Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = \pm x_2 \Rightarrow x_1 \neq x_2 \Rightarrow$ یک به یک نیست

جای x,y عوض، $f^{-1}: x = \frac{y}{\sqrt{y^2-5}} \Rightarrow x^2 = \frac{y^2}{y^2-5} \Rightarrow x^2y^2$

$-5x^2 = y^2 \Rightarrow y^2(x^2-1) = 5x^2 \Rightarrow y^2 = \frac{5x^2}{x^2-1}$

$\Rightarrow y = \pm\frac{\sqrt5\, x}{\sqrt{x^2-1}} \rightarrow$ تابع نیست ← عوض x,y با (+)

$f(x) \begin{cases} x^2-4x+k & x \geq 2 \Rightarrow -\frac{b}{2a} = 2 \\ x^2+2x+4k & x < 2 \Rightarrow \frac{-b}{2a} = -1 \end{cases}$ ۵-

$\rightarrow k-4 \geq 4k+8 \Rightarrow 3k \leq -12 \Rightarrow k \leq -4$

$f(x) = 3x - |x| \xrightarrow{\text{دوضابطه‌ای}} \begin{cases} 2x & x \geq 0 \\ 4x & x < 0 \end{cases}$ ۶-

$\Rightarrow f^{-1}(x) = \begin{cases} \frac{1}{2}x & x \geq 0 \\ \frac{1}{4}x & x < 0 \end{cases}$

$f^{-1}(x) = \frac{3x}{8} - \frac{\frac{1}{8}|x|}{1}$

$= \frac{15x}{40} - \frac{5}{40}|x|$

$\Rightarrow a = \frac{3}{8}, b = -\frac{1}{8} \Rightarrow ab = -\frac{3}{64}$

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$f(x) = \frac{3x+4}{x+m} \Rightarrow f(2) = -10 \Rightarrow 10m = -30 \Rightarrow m = -3$ ۷-

$\Rightarrow m = -3 \Rightarrow \underline{x \neq 3)}$, $f(-10) = 2 \Rightarrow f^{-1} = f$

$y = f \circ f^{-1}(x) + f^{-1}(x) = x + \frac{3x+4}{x-3} \Rightarrow x \neq 3) \quad \frac{3x+4}{x-3} = 0$

$\Rightarrow 3x+4 = 0 \Rightarrow \underline{x = -\frac{4}{3})}$

$f(3-2x) + 4g(\frac{x}{2}) \Rightarrow g(\frac{x}{2}) = 2 - f(3-2x) \Rightarrow g(x) = t$ ۸-

$\Rightarrow t = 2 - f(3-4x) \qquad f^{-1}(2-t) = 3-4x \Rightarrow f^{-1}(2-t)$

$g^{-1}(t) = x \qquad = 3 - 4g^{-1}(t)$

$\Rightarrow 4g^{-1}(t) = 3 - f^{-1}(2-t) \xrightarrow{t=4} 4g^{-1}(4) = 3 - f^{-1}(-2)$

$\Rightarrow 4g^{-1}(4) + f^{-1}(-2) = 3$

$y = \frac{5x-13}{1-x} \xrightarrow[\text{قرینه}]{x \leftrightarrow y} y = \frac{-5x-13}{1+x} \Rightarrow y = \frac{5x+13}{1+x}$ ۹-

$\xrightarrow{\text{۲ واحد}} \frac{5(x-2)+13}{1+(x-2)} = \frac{5x+3}{x-1} \xrightarrow[\text{پایین}]{\text{۳ واحد}} \frac{5x+3}{x-1} - 3$

$= \frac{5x+3-3x+3}{x-1} = \frac{2x+6}{x-1} \Rightarrow f(x) = \frac{2x+6}{x-1} \Rightarrow f^{-1}(x) : x = \frac{2y+6}{y-1}$

$\Rightarrow xy - x = 2y+6 \Rightarrow y(x-2) = 6+x \Rightarrow y = \frac{6+x}{x-2} = f^{-1}(x)$

$\frac{6+x}{x-2} = \frac{2x+6}{x-1} \longrightarrow x^2+5x-6 \longrightarrow (x-6)(x+1)$

$= 2x^2+2x-12 \longrightarrow x^2-3x-6=0 \longrightarrow -\frac{b}{a} = -\frac{(-3)}{1} = 3$

$f(x) = x+4[x] \xrightarrow{\text{ضابطه}} g(x) = f^{-1}(x) \longrightarrow x = y+4[y]$ ما

$\longrightarrow y = x-\frac{4}{5}[x] = g(x) \xrightarrow{\text{براکت از طرفین}} [y] = [\frac{1}{5}x+4[x]]$

$\longrightarrow [y] = \frac{1}{5}[x] \longrightarrow f(x)-g(x) = x+4[x]-x+\frac{4}{5}[x] = \frac{24}{5}[x]$