نام و نام خانوادگی ...طاها حق جویی... پاسخنامه تشریحی تکلیف شماره ۱۰ کلاس دوازده پسر A

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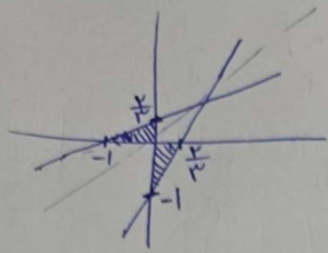

$y = \frac{3}{2}x - 1$

$S = \frac{\frac{2}{3} \times 1}{2} = \frac{1}{3}$

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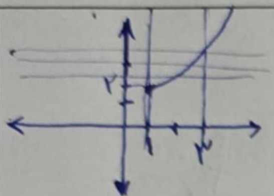

ضابطه معکوس هر دو: $y = (x-1)^2 + 2 \to y - 2 = (x-1)^2$ ← هر دو تابع یک به یک است ✓

$\to \sqrt{y-2} + 1 = x \to y = \boxed{\sqrt{x-2} + 1 = f^{-1}(x)} \to x \geq 2$ (I)

الف) $D_f = R_{f^{-1}} \to f^{-1}(x) = 1 \to \sqrt{x-2} + 1 = 1 \to x = 2 \to D_{f^{-1}} = [2, +\infty)$ (I) ✓

ب) $f^{-1}(x) = 3 \to \sqrt{x-2} + 1 = 3 \to x = 6 \to D_{f^{-1}} = [6, +\infty)$ (I) ✓

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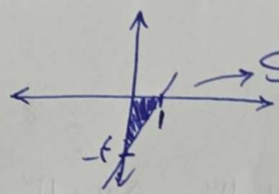
$f(x) = 2x - \sqrt{16 - 16x + 4x^2} = 2x - 2\sqrt{(x-2)^2} = 2x - 2|x-2|$

$= \begin{cases} x \geq 2: & 2x - 2x + 4 = 4 \\ x < 2: & 2x + 2x - 4 = 4x - 4 \end{cases}$ ← وارون پذیر ✓ ←



$S = \frac{1 \times 4}{2} = 2$

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$x^2 - 5 > 0 \to x < -\sqrt{5}$ یا $\sqrt{5} < x$

$y_1 = y_2 \to \frac{x_1}{\sqrt{x_1^2 - 5}} = \frac{x_2}{\sqrt{x_2^2 - 5}} \to \frac{x_1^2}{x_1^2 - 5} = \frac{x_2^2}{x_2^2 - 5}$

$x_2^2 x_1^2 - 5x_2^2 = x_1^2 x_2^2 - 5x_1^2 \to x_2^2 = x_1^2 \to x_1 = \pm x_2$! با توجه به دامنه یک به یک است ✓

$f^{-1}(x): y\sqrt{x^2 - 5} = x \to y^2(x^2 - 5) = x^2 \to y^2x^2 - x^2 = 5y^2 \to x^2(y^2 - 1) = 5y^2 \to x^2 = \frac{5y^2}{y^2 - 1}$

$\to x = \pm\sqrt{\frac{5y^2}{y^2 - 1}} \to f^{-1}(x) = \pm\sqrt{\frac{5x^2}{x^2 - 1}} \xrightarrow{x^2 - 1 > 0 \to |x| > 1} f^{-1}(x) = \begin{cases} \sqrt{\frac{5x^2}{x^2 - 1}} & x > 1 \\ -\sqrt{\frac{5x^2}{x^2 - 1}} & x < -1 \end{cases}$ ✓

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$x = 2 \to y_1 = 4 - 8 + k \to y_1 = k - 4$

$x = 2 \to y_2 = -4 + 12 + 3k \to y_2 = 3k + 8$

$y_1 \geq y_2 \to k - 4 \geq 3k + 8$

$\to 12 \geq 2k \to \boxed{6 \geq k}$

$y=3x+\|x\| \to y^2=9x^2+x^2+6x\|x\| \to f(x)=\begin{cases}4x & x\geq 0\\ 2x & x<0\end{cases} \to g(x)=\begin{cases}\frac{x}{4} & x\geq 0\\ \frac{x}{2} & x<0\end{cases}$ $=\begin{cases}(a+b)x & x\geq 0\\ (a-b)x & x<0\end{cases} \to \begin{matrix}a+b=\frac{1}{4}\\ a-b=\frac{1}{2}\end{matrix} \to 2a=\frac{3}{4} \to \boxed{a=\frac{3}{8}},\ \boxed{b=-\frac{1}{8}}$ $ab=-\frac{3}{64}$	۶
$f(2)=10 \to \frac{6+4}{2+m}=10 \to -20-10m=10 \to m=-3 \to f(x)=\frac{3x+4}{x-3} \xrightarrow{x\neq 3} f=f^{-1}$ $\to \cancel{f^{-1}\circ f(x)}+\cancel{f^{-1}(x)}$ $\cancel{x}+f(x)=\cancel{x} \to f(x)=0 \to \frac{3x+4}{x-3}=0 \to \boxed{x=-\frac{4}{3}}$	۷
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$\xrightarrow{\text{قرینه به مبدا}} -\frac{-5x-13}{1+x}=\frac{5x+13}{1+x} \xrightarrow[\text{راست}]{\text{۲ واحد به}} \frac{5(x-2)+13}{(x-2)+1}=\frac{5x+3}{x-1} \xrightarrow[\text{پایین}]{\text{۳ واحد}}$ $\frac{5x+3}{x-1}-3=\boxed{\frac{2x+6}{x-1}=f(x)}$ $f(x)=f^{-1}(x)$ $f^{-1}(x): y=\frac{2x+6}{x-1} \to yx-y=2x+6$ $\to yx-2x=6+y \to \boxed{f^{-1}(x)=\frac{x+6}{x-2}}$ $\quad \frac{2x+6}{x-1}=\frac{x+6}{x-2} \to x^2-3x-6=0$ [redacted] $\to x_1+x_2=3$	۹
$f^{-1}(x): x=y+4[y] \to y=x-4[y] \to [x]=[y+4[y]]=[5y]$ $\to [y]=[\frac{x}{5}] \to f^{-1}(x)=x-4[\frac{x}{5}]=g(x) \to (f-g)(x)=x+4[x]-x+4[\frac{x}{5}]$ $=4([x]+[\frac{x}{5}])$	۱۰

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