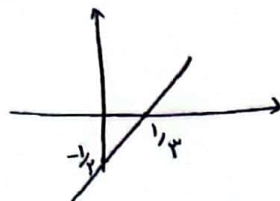


۲-۱

$$y = \frac{3x-1}{2} = \frac{3}{2}x - \frac{1}{2}$$



$$S = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6} \quad \checkmark$$

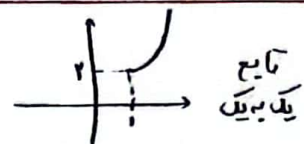
برخورد
باصفا
ما x

$$\frac{3}{2}x - \frac{1}{2} = 0 \rightarrow x = \frac{1}{3}$$

الف) $y = x^2 - 2x + 3, x \in [1, +\infty)$



$$x \geq 1$$



۲-۲

$$y = (x-1)^2 - 1 + 3 \Rightarrow \sqrt{y-2} = |x-1| \xrightarrow{x \geq 1} x = \sqrt{y-2} + 1 \rightarrow f^{-1}(y) = \sqrt{y-2} + 1 \quad D_{f^{-1}} = R_f \quad x \geq 1 \quad \checkmark$$

ب) $y = x^2 - 2x + 3, x \in [3, +\infty)$



$$x \geq 3$$



$$y = (x-1)^2 - 1 + 3 \Rightarrow \sqrt{y-2} = |x-1| \xrightarrow{x \geq 3} x = \sqrt{y-2} + 1 \rightarrow f^{-1}(y) = \sqrt{y-2} + 1 \quad D_{f^{-1}} = R_f \quad x \geq 3 \quad \checkmark$$

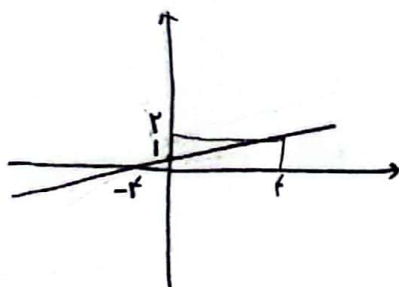
$$f(x) = 2x - \sqrt{17 - f(x)(f-x)} = 2x - \sqrt{f(f-x(f-x))} = 2x - \sqrt{f(f-fx+x^2)} = 2x - \sqrt{f(x-f)^2} \quad \checkmark$$

$$= 2x - 2|x-f| = \begin{cases} f & x \geq f \\ f-x & x < f \end{cases}$$

تابع در بازه
یک به یک
است.

$$f^{-1}(y) \equiv y = f-x-f \rightarrow x = \frac{y+f}{f}$$

$$\rightarrow f^{-1}(y) = \frac{y+f}{f} \quad D_{f^{-1}} = R_f \quad x \leq f \quad \checkmark$$



$$S = \frac{f \times f}{f} = f$$

$$S = \frac{(1+2) \times 2}{2} = 3$$

برای بررسی اینکه
یک بودن تابع

$$f(x_1) = f(x_2) \rightarrow x_1 = x_2$$

$$\frac{x_1}{\sqrt{x_1^2 - 5}} = \frac{x_2}{\sqrt{x_2^2 - 5}}$$

①
مخرج هر دو مثبت می
باشد و علامت
آنرا برای مساوی شدن
 $x_1, x_2 > 0$

۲

$$x_1 \sqrt{x_2^2 - 5} = x_2 \sqrt{x_1^2 - 5} \xrightarrow{||^2} x_1^2 (x_2^2 - 5) = x_2^2 (x_1^2 - 5) \rightarrow (x_1 x_2)^2 - 5x_1^2 = (x_1 x_2)^2 - 5x_2^2$$

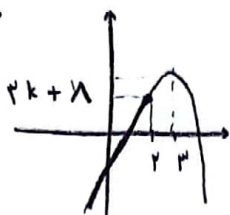
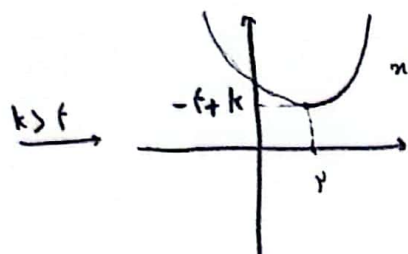
$$\rightarrow 5x_1^2 = 5x_2^2 \rightarrow x_1 = \pm x_2 \xrightarrow{①} x_1 = x_2 \quad \text{یک به یک است.}$$

$$y = \frac{x}{\sqrt{x^2 - 5}} \rightarrow y \sqrt{x^2 - 5} = x \xrightarrow{||^2} y^2 (x^2 - 5) = x^2 \rightarrow y^2 x^2 - 5y^2 = x^2 \rightarrow y^2 x^2 - x^2 = 5y^2$$

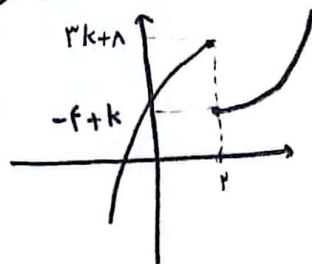
$$\rightarrow x^2 = \frac{5y^2}{y^2 - 1} \rightarrow x = \sqrt{\frac{5y^2}{y^2 - 1}} \rightarrow f^{-1}(y) = \sqrt{\frac{5y^2}{y^2 - 1}} \quad \checkmark$$

۵- برای حل این مسئله دو حالت در نظر می گیریم، $k > f$ و $k < f$ (در حل متوجه شویم چرا؟)

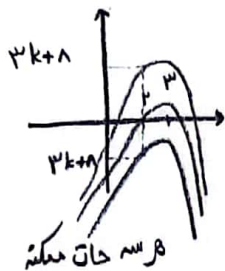
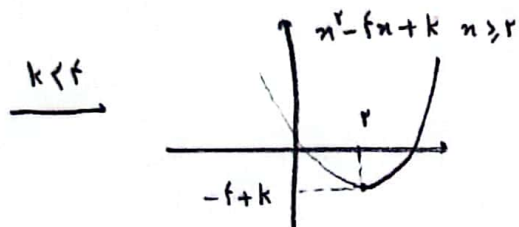
۱، ۷۵



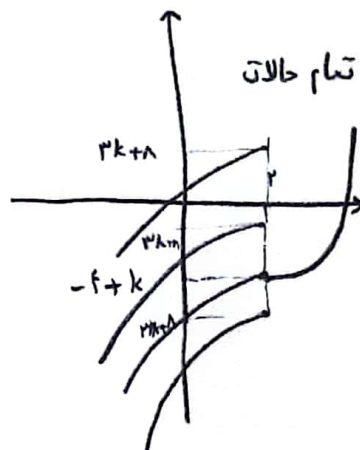
ترکیب



یک به یک
نی باشد



ترکیب



تمام حالات

$$3k + 8 < -f + k \rightarrow 2k < -12 \rightarrow k < -6$$

مساوی هم
نیستونه باشه!

برای
یک به یک
بودن

$$f(x) = rx + |x| \rightarrow \begin{cases} rx & x \geq 0 \\ rx + x & x < 0 \end{cases} \xrightarrow{f(x)} \begin{cases} \frac{x}{r} & x \geq 0 \\ \frac{x}{r} & x < 0 \end{cases} \rightarrow f^{-1}(x) = \begin{cases} \frac{x}{r} & x \geq 0 \\ \frac{x}{r} & x < 0 \end{cases} \quad -4$$

$$g(x) = f^{-1}(x) = ax + b|x| \begin{cases} x \geq 0 & ax + bx = \frac{x}{r} \rightarrow a + b = \frac{1}{r} \\ x < 0 & ax - bx = \frac{x}{r} \rightarrow a - b = \frac{1}{r} \end{cases} \rightarrow a = \frac{r}{2}, b = -\frac{1}{2} \rightarrow ab = -\frac{r}{4} \checkmark$$

$$f(x) = \frac{rx + f}{x + m} \xrightarrow{(r, -1)} \frac{y + f}{r + m} = -1 \rightarrow r + m = -1 \rightarrow m = -r \quad -v$$

$$f(x) = \frac{rx + f}{x - r} \xrightarrow{m=r} f(x) = f^{-1}(x)$$

$$y = \frac{f \circ f^{-1}(x) + f^{-1}(x)}{x} = \frac{x + f^{-1}(x)}{x} = x + \frac{rx + f}{x - r} \xrightarrow{y=x} \frac{rx + f}{x - r} + x = x \rightarrow \frac{rx + f}{x - r} = 0$$

$$\rightarrow x = \frac{-f}{r} \checkmark$$

$$f(r - rx_1) = r - g\left(\frac{x_1}{r}\right), \quad f g^{-1}(f) + f^{-1}(-r) = ? \quad -A$$

$$f(r - rx_1) = -r \rightarrow f^{-1}(-r) = r - rx_1 \quad \textcircled{1}$$

$$\xrightarrow{r x_1} f(r - rx_1) = r - g\left(\frac{x_1}{r}\right) \rightarrow g\left(\frac{x_1}{r}\right) = f \rightarrow g^{-1}(f) = \frac{x_1}{r} \quad \textcircled{2}$$

$$\xrightarrow{\textcircled{1} + \textcircled{2}} r - rx_1 + rx_1 = r \checkmark$$

$$y = \frac{\Delta x - 13}{1-x} \xrightarrow{\text{مرکز ثقل (0,1)}} -y = \frac{-\Delta x - 13}{1+x} \rightarrow y = \frac{\Delta x + 13}{1+x} \rightarrow y = \frac{\Delta(x-1) + 13}{1+x-2} - 3$$

$$= \frac{\Delta x + 3}{x-1} - 3 = \frac{\Delta x + 3 - 3x + 3}{x-1} = \frac{3x + 4}{x-1} = f(x)$$

$$f^{-1}(x) : y = \frac{3x+4}{x-1} \rightarrow yx - y = 3x + 4 \rightarrow yx - 3x = y + 4 \rightarrow f^{-1}(x) = \frac{x+4}{x-3}$$

$$f(x) = f^{-1}(x) \rightarrow \frac{3x+4}{x-1} = \frac{x+4}{x-3} \rightarrow (3x+4)(x-3) = (x-1)(x+4) \rightarrow 3x^2 + 3x - 12 = x^2 + 4x - 4$$

$$\rightarrow x^2 - 3x - 4 = 0 \rightarrow S = x_1 + x_2 = \frac{-b}{a} = 3 \quad \checkmark$$

$$f(x) = x + f[x]$$

$$f(x) - f^{-1}(x) = ? \quad D_{f \circ f^{-1}} = D_f \cap R_{f^{-1}} = \mathbb{R} \cap [-\infty, -5, -4) \cup [0, 1) \cup (2, 4) - \} = -\infty, (-5, -4) \cup [0, 1) \cup (2, 4)$$

$$f(x) = \begin{cases} x-2 & -5 \leq x < -4 \\ x & -4 \leq x < 0 \\ x+2 & 0 \leq x < 4 \end{cases}, \quad f^{-1}(x) = \begin{cases} \Delta(x+2) & -5 \leq x < -4 \\ \Delta(x+2) & -4 \leq x < 0 \\ \Delta(x+2) & 0 \leq x < 4 \end{cases} \rightarrow \begin{cases} x+2 & -5 \leq x < -4 \\ x & -4 \leq x < 0 \\ x-2 & 0 \leq x < 4 \end{cases}$$

$$\Rightarrow \begin{cases} -2f & -5 \leq x < -4 \\ 0 & -4 \leq x < 0 \\ 2f & 0 \leq x < 4 \end{cases}$$

بهی طرفت نکند و نه آید!

$$y = x + f[x] \xrightarrow{\text{از طرفین با یکسان ضربه بزنیم}} [y] = [x] + f[x] = \Delta[x] - [x] = \frac{[y]}{\Delta}$$

$$y = x + f \frac{[y]}{\Delta} \rightarrow x = y - f \frac{[y]}{\Delta} \xrightarrow{\text{ضرب}} y = x - f \frac{[x]}{\Delta}$$

$$(f - g)(x) = x + f[x] - x + f \frac{[x]}{\Delta} = f, \Delta[x]$$