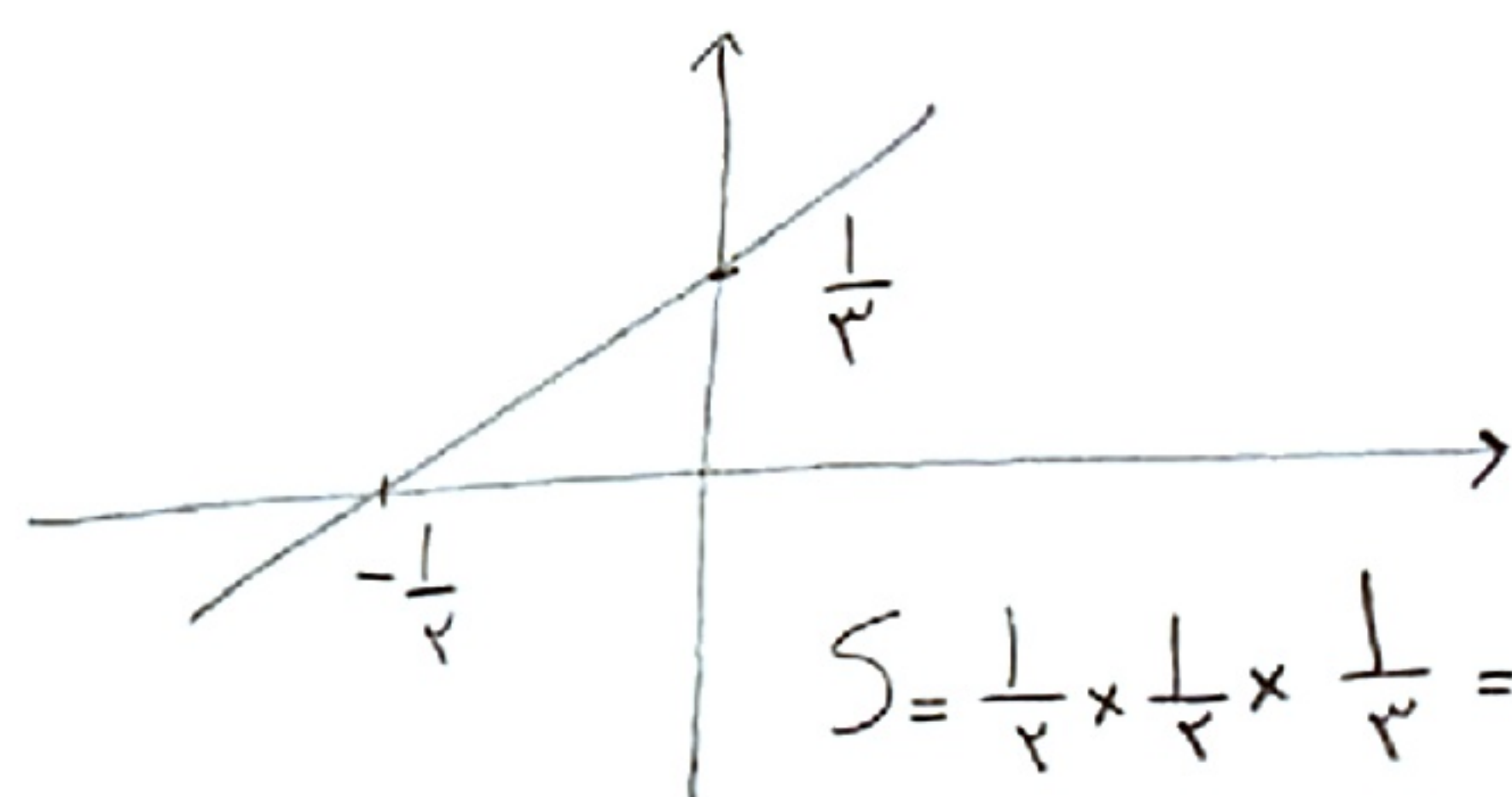


$$2y = 3x - 1 \rightarrow \text{دوین تابع: } 2x = 3y - 1 \rightarrow 2x + 1 = 3y$$

$$\frac{2x+1}{3} = y$$



$$x=0 \rightarrow y = \frac{1}{3}$$

$$y=0 \rightarrow 2x+1=0 \rightarrow x = -\frac{1}{2}$$

$$S = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{12} \checkmark$$

(۲)

۱

$$y = x^2 - 2x + 3 \quad x_s = \frac{-(-2)}{2(1)} = 1, \quad x \in [1, +\infty)$$

تابع یکتا نیست.

$$y = (x-1)^2 + 2 \rightarrow y-2 = (x-1)^2 \rightarrow \sqrt{y-2} = x-1 \rightarrow x = 1 + \sqrt{y-2}$$

$$f(x) = 1 + \sqrt{x-2}, \quad Df^{-1} = Rf = [2, +\infty) \checkmark$$

$$b) y = x^2 - 2x + 3 \quad x_s = 1, \quad x \in [2, +\infty)$$

$$f(x) = 1 + \sqrt{x-2}, \quad Df^{-1} = Rf = [2, +\infty) \checkmark$$

تابع یکتا نیست تابع یکتا نیست.

داریم تابع با وارون نامر در قسمت اول یکتا است.

(۲)

۲

$$f(x) = 2x - \sqrt{4-4x(x-x)} = 2x - \sqrt{4-4x^2 + 4x^2} = 2x - \sqrt{(2x-2)^2}$$

$$= 2x - |2x-2| \rightarrow \begin{cases} 4 & x \geq 2 \rightarrow \text{داریم یکتا} \\ 4x-4 & x \leq 2 \rightarrow \text{داریم یکتا} \end{cases}$$

$$f(x) = 4x-4 \rightarrow f^{-1}(x) = \frac{1}{4}x + 1, \quad x \in Df^{-1} = Rf = (-\infty, 4]$$

$$S = \frac{1}{2} (2+1) \times 4 = 6 \checkmark$$

(۲)

۳

$$f(x) = \frac{x}{\sqrt{x^2-8}} \quad f(x_1) = f(x_2) \rightarrow x_1 = x_2$$

$$\frac{x_1}{\sqrt{x_1^2-8}} = \frac{x_2}{\sqrt{x_2^2-8}} \rightarrow \frac{x_1^2}{x_1^2-8} = \frac{x_2^2}{x_2^2-8} \rightarrow x_1^2 x_2^2 - 8x_1^2 = x_1^2 x_2^2 - 8x_2^2 \rightarrow x_1^2 = x_2^2 \rightarrow x_1 = \pm x_2$$

$$x_1^2 = x_2^2 \rightarrow x_1 = \pm x_2 \rightarrow \text{تابع دوین نیست است} \rightarrow x_1 = x_2 \checkmark$$

$$y = \frac{x}{\sqrt{x^2-8}} \rightarrow y^2 = \frac{x^2}{x^2-8} \rightarrow x^2 = \frac{8y^2}{y^2-1} \rightarrow x = \pm \frac{\sqrt{8}y}{\sqrt{y^2-1}} \rightarrow f^{-1}(y) = \frac{\sqrt{8}y}{\sqrt{y^2-1}} \checkmark$$

(۲)

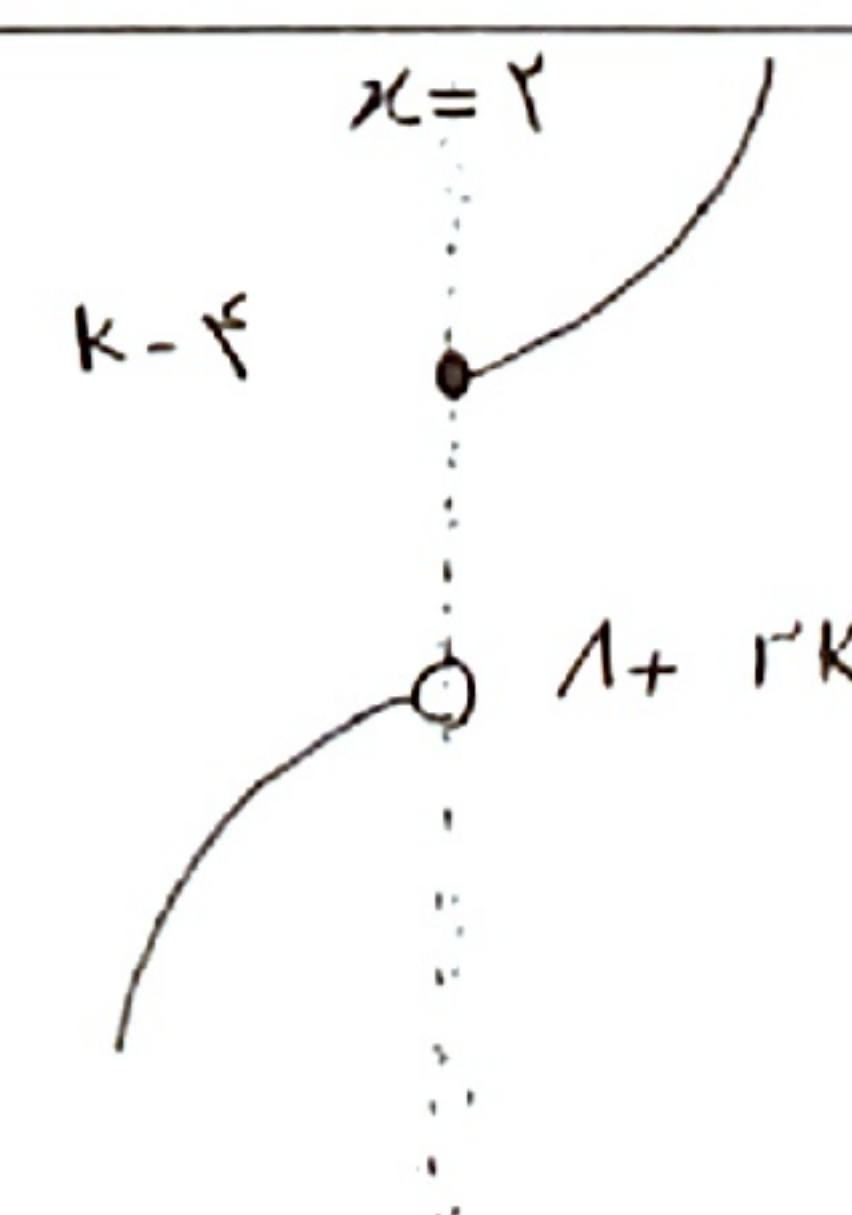
۴

$$g(x) = \begin{cases} x^2 - 4x + k & x \geq 2 \\ -x^2 + 4x + 3k & x < 2 \end{cases}$$

$$f(2) = 4 - 8 + k = k - 4$$

$$x_{s1} = \frac{-(-4)}{2} = 2 \rightarrow 1 + 3k \leq k - 4$$

$$x_{s2} = \frac{-14}{-2} = 7 \rightarrow 2k \leq -12 \rightarrow k \leq -6 \checkmark$$



(۲)

۵

$f(x) = rx + x \rightarrow f(x) \begin{cases} rx & ; x \geq 0 \\ rx & ; x < 0 \end{cases} \quad f'(x) \begin{cases} \frac{1}{r}x & ; x \geq 0 \\ \frac{1}{r}x & ; x < 0 \end{cases}$ $f^{-1}(x) = g(x) = ax + b x \quad \begin{cases} (a+b)x & x \geq 0 \\ (a-b)x & x < 0 \end{cases} \rightarrow \begin{cases} a+b = \frac{1}{r} \\ a-b = \frac{1}{r} \end{cases} \rightarrow \begin{cases} ra = \frac{r}{r} \\ a = \frac{r}{\lambda} \checkmark \\ b = -\frac{1}{\lambda} \end{cases}$ $a \cdot b = -\frac{1}{\lambda} \times \frac{r}{\lambda} = -\frac{r}{\lambda^2} \checkmark$	2 6
$f(x) = \frac{rx + \varepsilon}{x + m} \rightarrow f(r) = -1 \rightarrow \frac{r(r) + \varepsilon}{r + m} = -1 \rightarrow -r - 10m = 10$ $-10m = r$ $m = -r$ $f(x) = \frac{\frac{a}{r}x + \varepsilon}{x - r} \rightarrow f^{-1}(x) = f(x) \quad y = f \circ f^{-1}(x) = f(x)$ $a + d = 0$ $y = x + f(x) = x + \frac{rx + \varepsilon}{x - r}$ $x \in Df^{-1} = Rf = R - \{r\} \rightarrow x = x + f(x) \rightarrow f(x) = 0$ $\frac{rx + \varepsilon}{x - r} = 0 \rightarrow rx + \varepsilon = 0 \rightarrow x = -\frac{\varepsilon}{r} \checkmark$	2 7
$\underbrace{f^{-1}(\varepsilon)}_a + \underbrace{f^{-1}(-r)}_b = r(a) + (b) = r \checkmark$ $g(a) = \varepsilon, f(b) = -r$ $f(r - rx) = r - g\left(\frac{x}{r}\right) \rightarrow f(b) = r - g(b) \rightarrow -r = r - \varepsilon \rightarrow -r = -r$ $r - rx = b \rightarrow \frac{r-b}{r} = x \rightarrow x = x \rightarrow ra = \frac{r-b}{r} \rightarrow \varepsilon a = r - b$ $\frac{rx}{r} = a \rightarrow x = ra$ $\underbrace{f^{-1}(\varepsilon)}_a + \underbrace{f^{-1}(-r)}_b = \varepsilon a + b = r \quad \varepsilon a + b = r$	2 8
$y = \frac{8x - 13}{1 - x} \xrightarrow{\text{تبدیل متغیر}} - \frac{8(-x) - 13}{1 - (-x)} = \frac{8x + 13}{1 + x} \xrightarrow{\text{تبدیل متغیر}} \frac{8(x-r) + 13}{1 + (x-r)}$ $= \frac{8x + r}{x - 1} \xrightarrow{\text{تبدیل متغیر}} \frac{8x + r}{x - 1} - r = \frac{rx + \varepsilon}{x - 1} \rightarrow f(x) = \frac{rx + \varepsilon}{x - 1}$ $f^{-1}(x) = \frac{x + \varepsilon}{x - r} \rightarrow f^{-1}(x) = f(x) \rightarrow \frac{x + \varepsilon}{x - r} = \frac{rx + \varepsilon}{x - 1}$ $x^2 + 8x - \varepsilon = rx^2 + rx - 13 \rightarrow x^2 - rx - \varepsilon = 0$ $\varepsilon = \frac{-(-r)}{1} = r \checkmark$	2 9
$f(x) = x + \varepsilon[x] \rightarrow f^{-1}(x) \rightarrow x = y + \varepsilon[y] \rightarrow [x] = [y + \varepsilon[y]]$ $[x] = \delta[y] \rightarrow \frac{1}{\delta}[x] = [y]$ $f^{-1}(x) = g(x) = x - \frac{\varepsilon}{\delta}[x]$ $(f - g)(x) = x + \varepsilon[x] - (x - \frac{\varepsilon}{\delta}[x]) = \frac{\varepsilon}{\delta}[x] \checkmark$ $D(f - g) = Df \cap Dg; Df = R, Dg = Df^{-1} = Rf$	2 10