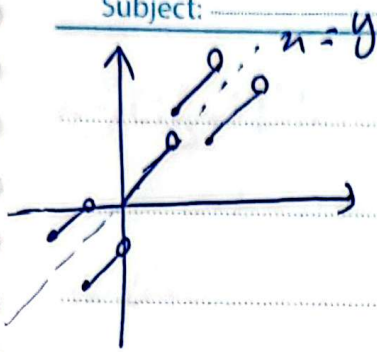


Subject:

Date:



الف) $n \in [0, 1] \Rightarrow$ ی. شمار

①

ب) $f \circ f^{-1}(m) = m - \frac{3}{4} \Rightarrow f \circ f^{-1}(m) = m, m \in \mathbb{R}$

$\mathbb{R} \quad f(m) = [-\frac{3}{4}, -1) \cup [0, 1) \cup \dots$

ج. ندارد. $\frac{3}{4}m - \frac{3}{4} \sim m = \frac{3}{4} \Rightarrow$

② محل برخورد معانبات افقی $(\frac{a}{c}) =$ برخورد معانبات قائم $(-\frac{b}{c}) =$ تابع وارونگی

$$\frac{a}{c} = -\frac{b}{c} \Rightarrow a = -b \sim y = \frac{a \cdot m + b}{c \cdot m + d} \xrightarrow{*} a + d = 0$$

$$f(m) = f(m)^{-1} = \frac{-b \cdot m + 3}{c \cdot m + b} \rightarrow f \circ f^{-1}(a - \sqrt{9}) = \underbrace{f \circ f^{-1}}_m(a - \sqrt{9}) = \boxed{a - \sqrt{9}}$$

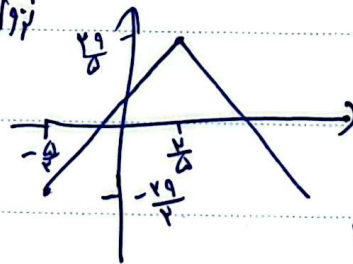
③ $m = 1 \Rightarrow$ فرض $m \neq -\frac{3}{4} \Leftarrow m + \frac{3}{4} \neq -m \Leftarrow$ وارونگی ندارد $f(m)$

$$f(m) = \frac{m+3}{m+4} \Rightarrow \frac{m+3}{m+4} = \frac{-1}{m-1} \Rightarrow m^2 + 3m - 3 = -m - 4 \Rightarrow m^2 + 4m + 1 = 0$$

$$\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = \frac{c}{p} = \frac{-3}{1} = \boxed{-3}$$

$$f^{-1}(m) \Rightarrow y = \frac{m+3}{m+4} \Rightarrow y \cdot m + 4 = m + 3 \xrightarrow{m=y} m \cdot y + 4 = y + 3 \Rightarrow y \cdot m = -1 \Rightarrow m = \frac{-1}{y}$$

$$\begin{cases} m > \frac{3}{4} \Rightarrow 3m + 4 > 0 \\ \frac{3}{4} > m > -\frac{3}{4} \Rightarrow 4m + 3 > 0 \\ m < -\frac{3}{4} \Rightarrow 3m - 4 < 0 \end{cases}$$



$$f(m) = \frac{3}{4} + \frac{1}{m}$$

$$f^{-1}(m) = y = \frac{-m+3}{m}$$

$$Df^{-1} = Rf = (-\infty, \frac{3}{4}]$$

$$g(m) = t = 2f(m) - 1 \rightarrow g(m) = t \rightarrow g(t^{-1}) = m$$

$$f(\frac{t+1}{2}) = \frac{t+1}{2} \rightarrow f^{-1}(\frac{t+1}{2}) = m \Rightarrow g(t)^{-1} = f(\frac{t+1}{2})^{-1} \rightarrow \begin{cases} b=1 \\ c=2 \\ a=\frac{1}{2} \\ d=0 \end{cases}$$

$$\frac{a \cdot c + d}{2 \cdot b} = \frac{2 \times \frac{1}{2} + 0}{2 \times 1} = \frac{1}{2} \Rightarrow \boxed{\frac{1}{2}}$$

$$f(m) = m + 2\sqrt{m} \quad ; \quad m \geq 0 \quad \Rightarrow [0, +\infty)$$

④

$$f^{-1}(m): y = m + 2\sqrt{m} \xrightarrow{m \leftrightarrow y} m = y + 2\sqrt{y} \Rightarrow m = (\sqrt{y} + 1)^2 - 1 \Rightarrow (\sqrt{y} + 1)^2 = m + 1$$

$$y = m + 2\sqrt{m} \Leftrightarrow y = m + 1 + 1 - 2\sqrt{m+1} \xrightarrow{\text{طرفین را به توان 2 می‌رسانیم}} \sqrt{y} = \sqrt{m+1} - 1 \leftarrow \sqrt{y} + 1 = \sqrt{m+1}$$

$$Df^{-1} = Rf = [0, +\infty)$$

$$f(m): y = m - 1 + 2\sqrt{m} \xrightarrow{\text{تبدیل متغیر}} m = y + 1 - 2\sqrt{y} \Rightarrow m = (\sqrt{y} - 1)^2 - 1 \Rightarrow (\sqrt{y} - 1)^2 = m + 1$$

⑤

$$f^{-1}(m): m = y - 1 + 2\sqrt{y} \quad m - 1 + 2\sqrt{y} = m \Rightarrow 2\sqrt{y} = 1 \Rightarrow \boxed{m = 0} \quad \text{فقط این مقدار}$$

$$f(m) = m^2 + 4m \xrightarrow{\text{تبدیل متغیر}} f^{-1}(m) = m = y^2 + 4y \Rightarrow m = y(y + 4)$$

①

$$f^{-1}(m) > m \rightarrow f(m) < m \rightarrow m^2 + 4m < m \rightarrow m(m + 3) < 0 \Rightarrow \boxed{m < -3} \quad \text{فقط در این بازه}$$

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$$f(m) = -m + \sqrt{m+4} \quad ; \quad m \geq -4 \quad \xrightarrow{\text{تبدیل متغیر}} f^{-1}(m) \xrightarrow{\text{تبدیل متغیر}} f^{-1}(m+4) = g(m) = m - 4$$

⑨

$$y + 4 = \sqrt{m+4} \leftarrow m + 4 = -m - 4 + \sqrt{m+4} \leftarrow f(m-4) = m+4$$

$$y + 4 = \sqrt{m+4} \xrightarrow{\text{طرفین را به توان 2 می‌رسانیم}} m^2 + 4m = m + 4 \Rightarrow m = 0 \quad \text{پس جواب } \boxed{m = 0}$$

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$$\sqrt{m} + \sqrt{y} = 2 \Rightarrow f(m) = f^{-1}(m) \Rightarrow f \circ f(m) = f \circ f^{-1}(m) = m \quad ; \quad m \in Df^{-1} \quad \text{⑩}$$

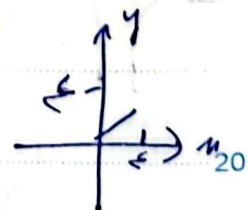
$$\sqrt{y} = 2 - \sqrt{m} \Rightarrow m \geq 0 \Rightarrow m \leq 4$$

$$\Rightarrow \boxed{0 \leq m \leq 4}$$



$$\sqrt{m} = 2 - \sqrt{y} \rightarrow y \geq 0 \rightarrow 2 - \sqrt{y} \geq 0$$

$$\rightarrow 2 - \sqrt{y} \geq 0 \rightarrow \sqrt{y} \leq 2 \rightarrow y \leq 4$$



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