

$$f(x) = x + [x] = y \rightarrow [y] = 2[x] \rightarrow \frac{[y]}{2} = [x]$$

$$x + \frac{[y]}{2} = y \rightarrow x = y - \frac{[y]}{2} \rightarrow f^{-1}(x) = x - \frac{[x]}{2}$$

الف) $f(x) = f^{-1}(x)$ (معادله را برای x حل می‌کنیم)

$$x + [x] = x \rightarrow [x] = 0 \rightarrow x \in [0, 1)$$

ب) $f \circ f^{-1}(x) = x = 2x - \frac{[x]}{2} \rightarrow x = \frac{[x]}{2} \rightarrow$ جواب ندارد

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بروز مطالب افق و قائم روی نیمه‌ها نامی است $f(x) = f^{-1}(x)$

$$f(x) = \frac{ax+3}{x+b} \rightarrow f^{-1}(x) = \frac{-bx+3}{x-a}$$

$$\frac{ax+3}{x+b} = \frac{-bx+3}{x-a} \rightarrow b = -a \rightarrow f(x) = f^{-1}(x)$$

$$f \circ f^{-1}(5 - \sqrt{2}) = f \circ f^{-1}(5 - \sqrt{2}) = 5 - \sqrt{2}$$

$$f(x) = \frac{mx+3}{x+m+3} \rightarrow f^{-1}(x) = \frac{-(m+3)x+3}{x-m} \rightarrow \frac{mx+3}{x+m+3} = \frac{-(m+3)x+3}{x-m}$$

$$\rightarrow (-m-3)x^2 + (-m^2-3m)x - 9x + 3m+9 = 0$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{m^2+3m}{-m-3}}{\frac{3m+9}{-m-3}} = \frac{m^2+3m}{3m+9} = \frac{m}{3} \quad \Delta > 0 \rightarrow (m^2+3m+9)^2 - 4(-m-3)(-m^2-3m+9) > 0$$

عبارت معادله را برای x حل می‌کنیم $\rightarrow \frac{mx+3}{x+m+3} = x \rightarrow mx+3 = x^2 + (m+3)x \rightarrow x^2 + 3x - 3 = 0$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{1-3}{-3} = 1$$

$$f(x) = |2x+5| - |5x-2|$$

$$f(x) = \begin{cases} 3x-7 & x \leq -\frac{5}{2} \\ 7x+3 & -\frac{5}{2} < x < \frac{2}{5} \\ -3x+7 & x \geq \frac{2}{5} \end{cases}$$

$$f(x) = -3x+7 \quad \frac{2}{5} \leq x$$

$$f^{-1}(x) = \frac{7-x}{3} \quad x \leq \frac{29}{5}$$

$$g(x) = 2f(3x) - 1 = y \rightarrow g^{-1}(x) = \frac{1}{3} f^{-1}\left(\frac{x+1}{2}\right) + 0$$

$$g^{-1}(x) = a f^{-1}\left(\frac{x+b}{c}\right) + d$$

$$a = \frac{1}{3} \quad b = 1 \quad c = 2 \quad d = 0$$

$$\frac{ac+d}{2b} = \frac{\frac{1}{3} \times 2 + 0}{2 \times 1} = \frac{1}{3}$$

$$f(x) = (\sqrt{x+1})^2 - 1 = y \rightarrow f^{-1}(x) = (\sqrt{x+1} - 1)^2$$

$$D_f = [0, +\infty) \rightarrow R_{f^{-1}} = [0, +\infty)$$

$$R_f = [0, +\infty) \rightarrow D_{f^{-1}} = [0, +\infty)$$

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$$y^x \rightarrow \text{صعودی آید}$$

$$x-1 \rightarrow \text{صعودی آید}$$

$$f(x) = x-1 + y^x \rightarrow \text{صعودی آید}$$

همه صعودی آید - قطع کردن وارون در $y=x$ یک نقطه تلاقی

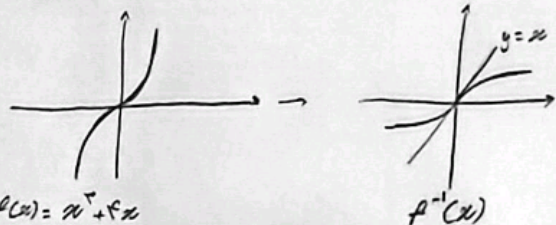
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$$f^{-1}(x) \geq x \rightarrow f(x) \leq x \rightarrow x^3 + 4x \leq x \rightarrow x^3 + 3x \leq 0 \rightarrow x(x^2 + 3) \leq 0$$

$$\frac{-\infty}{-} \quad \frac{0}{0} \quad \frac{+\infty}{+}$$

$$D_f = (-\infty, 0] \rightarrow \text{بی نهایت منفی}$$

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$$y = \sqrt{x+4} - x \rightarrow x = \sqrt{y+4} - y \rightarrow x+4 = \sqrt{y+4} - y \rightarrow x = \sqrt{y+4} - (y+4)$$

$$y = x-3 \rightarrow x = y+3$$

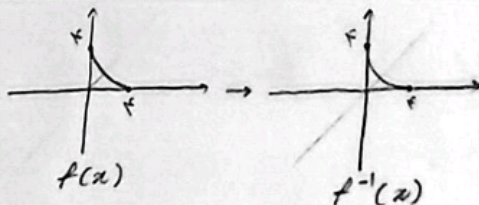
$$\sqrt{y+4} - (y+4) = y+3 \rightarrow \sqrt{y+4} = 2y+7 \rightarrow y+4 = 4y^2 + 28y + 49 \rightarrow 4y^2 + 24y + 45 = 0 \rightarrow y^2 + 6y + 11.25 = 0 \rightarrow (y+1.25)(y+9) = 0 \rightarrow \begin{cases} y = -3 \rightarrow x = 0 \\ y = -\frac{15}{4} \rightarrow x = -\frac{9}{4} \end{cases}$$

در نمودار

$$\sqrt{x} + \sqrt{y} = 2$$

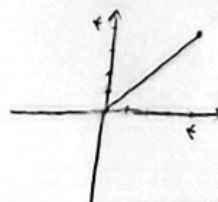
$$D_f = [0, 4]$$

$$R_f = [0, 4]$$



$$f(x) = f^{-1}(x)$$

$$f \circ f(x) = f^{-1} \circ f(x) = x \quad D_{f \circ f} = [0, 4] \rightarrow$$



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