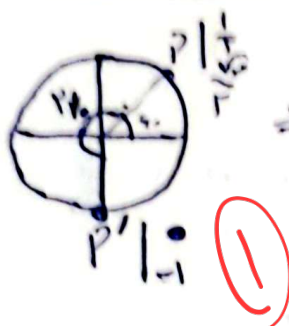


$$= \frac{-\frac{a}{r} - \frac{b}{r}}{-\frac{a}{r} + \frac{b}{r}} \Rightarrow \frac{\sqrt{r}}{r} \Rightarrow \frac{b+a}{b-a} = \frac{1}{r} \Rightarrow rb + ra = b - a$$

$$rb = -ra \Rightarrow \frac{b}{a} = -r \quad \checkmark$$

(2)



$$PP' = \sqrt{R^2 + R^2 - 2R^2 \cos \theta}$$

$$PP' = \sqrt{r^2 + r^2 - 2r^2 \cos \frac{\pi}{3}} = \sqrt{r^2 + r^2 - 2r^2 \cdot \frac{1}{2}} = \sqrt{r^2 + r^2 - r^2} = \sqrt{r^2} = r$$

$$\Rightarrow POP' = \sqrt{r^2 + r^2} = \sqrt{2}r$$

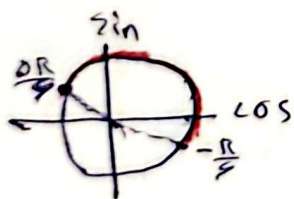
$$\frac{\pi}{9} = \frac{\alpha}{180} \Rightarrow \frac{1}{9} = \frac{\alpha}{180} \Rightarrow \alpha = 20^\circ \Rightarrow \text{مقیاس} = 40^\circ \Rightarrow 20^\circ \quad \checkmark$$

(2)

$$x = \left| \frac{1}{r} M - 30^\circ \right| \Rightarrow 20^\circ = \left| \frac{1}{r} M - 90^\circ \right| \Rightarrow M = \frac{140}{11} \quad \checkmark$$

$$90^\circ - \frac{1}{r} M = 20^\circ \Rightarrow \frac{1}{r} M = 70^\circ \Rightarrow M = \frac{140}{11} \quad \checkmark$$

(2)



$$-\frac{1}{r} < \frac{m-3}{2} \leq 1 \Rightarrow -\frac{1}{r} < m-3 \leq 2 \Rightarrow \frac{1}{r} < m \leq 5 \quad \checkmark$$

(2)

① $\sin(\omega) = \sin(\pi + \omega) = -\sin(\omega)$ ② $\cos(\omega) = \cos(\pi - \omega) = -\cos(\omega)$ ③ $\sin(\pi - \omega) = \sin(\omega)$ ④ $\cos(\pi + \omega) = -\cos(\omega)$ ⑤ $\sin(\pi + \omega) = -\sin(\omega)$

$$\frac{\cos(\omega) - k \sin(\omega)}{-\cos(\omega) + \sin(\omega)} = r \Rightarrow \frac{1 - k \tan(\omega)}{-1 + \tan(\omega)} = r \Rightarrow \frac{1 - \frac{1}{2}k}{-1 + 1} = r \Rightarrow \frac{1 - \frac{1}{2}k}{0} = r \Rightarrow \frac{1}{2}k + 1 = -r \Rightarrow \frac{1}{2}k = -1 - r \Rightarrow k = -2 - 2r$$

$\frac{1}{2}k = -\frac{11}{2} \Rightarrow k = -11$ ✓

① $\tan(\pi - x) = -\tan(x)$ ② $\cot(\pi - x) = -\cot(x)$ ③ $\cot(\pi + x) = \cot(x)$ ④ $\tan(\pi + x) = \tan(x)$ ⑤ $\cot(\pi + x) = \cot(x)$

$$\Rightarrow \frac{\cot(x) + \cot(x)}{\tan(x) - \tan(x)} = \frac{r \cot(x)}{-\tan(x)} = \frac{r \cos(x)}{\sin(x)} = -r \frac{\cos(x)}{\sin(x)} = -r \cot(x)$$

$$\Rightarrow \frac{1 - \cos(x)}{1 + \cos(x)} = r \Rightarrow r + r \cos(x) = 1 - \cos(x) \Rightarrow r \cos(x) = 1 - r - \cos(x) \Rightarrow \cos(x) = \frac{1-r}{r+1} \Rightarrow \sin(x) = \sqrt{1 - \left(\frac{1-r}{r+1}\right)^2} = \frac{2r}{r^2+1}$$

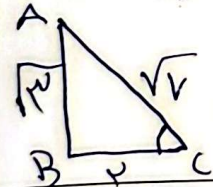
$$\Rightarrow -r \cot(x) = -r \frac{\cos(x)}{\sin(x)} = -r \frac{1-r}{2r} = -\frac{1-r}{2}$$

$$\frac{1}{\sin(x)} = \frac{r}{\pi} \Rightarrow x = \frac{1}{\sin(x)} \pi = \frac{r}{\pi} \pi \Rightarrow \frac{r}{\pi} \pi = \frac{r}{\pi} \pi \Rightarrow r_A = \frac{r}{\pi} \pi \Rightarrow r_A = \frac{r}{\pi} \pi$$

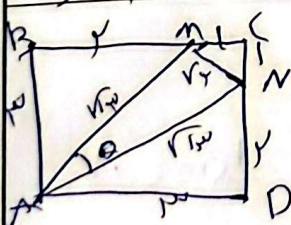
$$\frac{S_A}{S_B} = \frac{r_A^2}{r_B^2} = \left(\frac{r_A}{r_B}\right)^2 = \left(\frac{r}{r}\right)^2 = 1$$

① $\tan(x) = \frac{1}{\sqrt{2}} \Rightarrow \frac{BD}{AB} = \frac{1}{\sqrt{2}} \Rightarrow AB = \sqrt{2} BD$ ② $\sin(x) = \frac{1}{\sqrt{2}} \Rightarrow AD = BD$

③ $S_{ABC} = \frac{AB \times BC}{2} = \frac{\sqrt{2} \times BC}{2} = \sqrt{2} \Rightarrow BC = \sqrt{2} \Rightarrow DC = 1 \Rightarrow AC = \sqrt{AD^2 + DC^2} = \sqrt{2}$



$$\Rightarrow \sin(\angle C) = \frac{AB}{AC} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$



$AM = \sqrt{1 + \frac{1}{4}} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$ $MN = \sqrt{\frac{1}{4} + \frac{1}{4}} = \frac{\sqrt{2}}{2}$
 $AN = \sqrt{1 + \frac{1}{4}} = \frac{\sqrt{5}}{2}$

$$S_{AMN} = S_{ABCD} - S_{ABM} - S_{ADN} - S_{MNC} = 1 - \frac{1}{4} - \frac{1}{4} - \frac{1}{4} = \frac{1}{4}$$

$$\Rightarrow S_{AMN} = 1 - \frac{1}{4} = \frac{3}{4} \Rightarrow S_{AMN} = \frac{1}{2} AM \times AN \times \sin(\theta) = \frac{1}{2} \times \frac{\sqrt{5}}{2} \times \frac{\sqrt{5}}{2} \times \sin(\theta) = \frac{5}{8} \sin(\theta)$$

$$\Rightarrow \frac{3}{4} = \frac{5}{8} \sin(\theta) \Rightarrow \sin(\theta) = \frac{6}{5}$$