۲۵

دوازدهم ریاضی

-۱

$\sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$ $\quad$ $\sin\frac{11\pi}{6} = -\frac{1}{2}$

$\cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$ $\quad$ $\cos\frac{\pi}{3} = \frac{1}{2}$

$\tan\frac{5\pi}{6} = -\frac{\sqrt{3}}{3}$

$$\Rightarrow \frac{-\frac{\sqrt{3}}{2}(\alpha+\beta)}{\frac{1}{2}(\beta-\alpha)} = -\frac{\sqrt{3}}{3} \rightsquigarrow 3\alpha+3\beta = -\alpha+\beta$$

$$4\alpha = -2\beta \rightsquigarrow \beta = -2\alpha \rightsquigarrow \frac{\beta}{\alpha} = \boxed{-2}$$

-۲

$$\text{محیط} = 1+1+\sqrt{\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow 2+\left(\frac{1+\sqrt{3}}{2}\right)\sqrt{2} = 2+\frac{\sqrt{2}+\sqrt{6}}{2}$$

$$\Rightarrow \frac{4+\sqrt{2}+\sqrt{6}}{2}$$

-۳

$$\frac{D}{180} = \frac{Rad}{\pi} \rightarrow \frac{x}{180} = \frac{\frac{\pi}{9}}{\pi} \Rightarrow D = 20^\circ$$

به ازای هر ساعت شمار ۲۰° بچرخد => دقیقه شمار ۴۰ رفته

دقیقه	درجه
60	30
?	20

$\Rightarrow ? = 40\,min$

$\Rightarrow \boxed{2:40'}$

-۴

$$|5.5m - 30h| \rightsquigarrow |5.5m - 900| = 20$$

$$\rightsquigarrow 5.5m - 900 = 20 \rightarrow m = 20$$

$$\rightsquigarrow 5.5m - 900 = -20 \rightarrow m = \frac{160}{11} \simeq 12,\ldots$$

=> بعد از ۲۰ دقیقه برای دومین بار زاویه برابر ۲۰° خواهد شد...

-۵

$$-\frac{\pi}{6} < x \leq \frac{5\pi}{6} \rightsquigarrow -\frac{1}{2} < \sin x \leq 1$$

$$-\frac{1}{2} < \frac{m-3}{5} \leq 1 \rightsquigarrow -\frac{5}{2} < m-3 \leq 5$$

$$\frac{1}{2} < m \leq 8 \rightsquigarrow m \in \{1,2,3,4,5,6,7,8\} \rightarrow$$ ۸ عدد صحیح

-۶

$$\frac{\sin(90+15)+k\cos(270-15)}{\cos(180-15)+2\sin(180+15)} = 3$$

$$\rightsquigarrow \frac{\cos 15 - k\sin 15}{-\cos 15 - 2\sin 15} \xrightarrow{\div \cos 15} \frac{1-k\tan 15}{-1-2\tan 15}$$

$$\Rightarrow \frac{1-0.25k}{-1-0.5} = 3 \rightsquigarrow 1-0.25k = -4.5$$

$$\boxed{k = 22}$$

-۷

$$1-\cos x = 2+2\cos x \rightarrow 3\cos x = -1 \Rightarrow \cos x = -\frac{1}{3}$$

$$\sin^2+\cos^2 = 1 \rightarrow \sin^2 = 1-\frac{1}{9} \rightsquigarrow \sin = \frac{2\sqrt{2}}{3}$$ => ناحیه دوم سینوس مثبت

$$\frac{\tan(\frac{\pi}{2}-x)-\cos(\pi-x)}{\cos(\frac{3\pi}{2}-x)+2\tan(2\pi-x)} = \frac{\cot x+\cos x}{\tan b - 2\tan x}$$

$$\frac{2\cot x}{-\tan x} \rightsquigarrow \frac{2\left(-\frac{1}{2\sqrt{2}}\right)}{2\sqrt{2}} = \boxed{-\frac{1}{4}}$$

$$\tan = \frac{\frac{2\sqrt{2}}{3}}{-\frac{1}{3}} = -2\sqrt{2} \rightsquigarrow \cot x = -\frac{1}{2\sqrt{2}}$$

-۸

$$\frac{D}{180} = \frac{Rad}{\pi} \rightsquigarrow \frac{108}{180} = \frac{Rad}{\pi} \rightsquigarrow Rad = \frac{3\pi}{5}$$

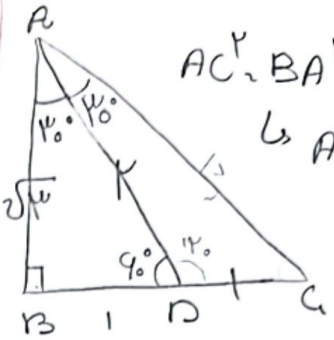
$$\frac{3\pi}{5} r_A = \frac{9\pi}{10} r_B \rightsquigarrow r_A = \frac{3}{2} r_B$$

$L = \alpha r$

$$\Rightarrow \frac{L_A}{L_B} = \left(\frac{r_A}{r_B}\right)^2 \rightsquigarrow \left(\frac{\frac{3}{2}r_B}{r_B}\right)^2$$

$$\Rightarrow \boxed{\frac{9}{4}}$$

-۹



$AC^۲ = BA^۲ + BC^۲$

$\hookrightarrow AC = \sqrt{۷}$

(۲)

$\angle ۳۰^\circ = \frac{۱}{۲} \Rightarrow AD = ۲$

$\rightarrow AB^۲ + BD^۲ = AD^۲ \rightsquigarrow AB = \sqrt{۴-۱} = \sqrt{۳}$

$S_{ABC} = \frac{۱}{۲} \times \sqrt{۳} \times x \times \sin ۹۰^\circ = \sqrt{۳}$

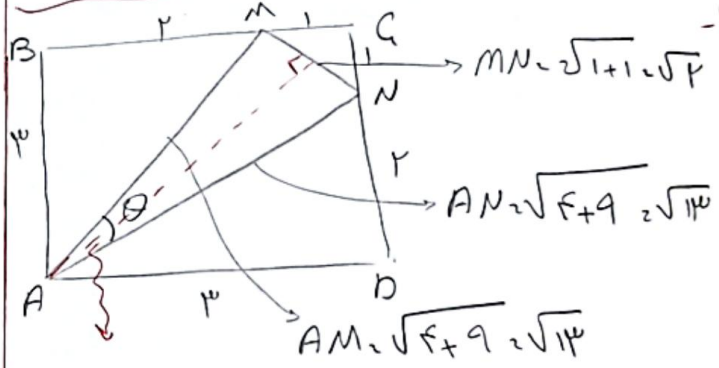
$x = ۲ \Rightarrow DC = ۱$

$\angle C = \sqrt{\frac{۳}{۷}}$, $\cos C = \frac{۲}{\sqrt{۷}}$

$\angle AC = ۲\angle C \cos C \rightarrow ۲ \times \frac{\sqrt{۳}}{\sqrt{۷}} \times \frac{۲}{\sqrt{۷}}$

$= \frac{۴\sqrt{۳}}{۷}$ ✓

-۱۰



$MN = \sqrt{۱+۱} = \sqrt{۲}$

$AN = \sqrt{۴+۹} = \sqrt{۱۳}$

$AM = \sqrt{۴+۹} = \sqrt{۱۳}$

$\angle \frac{\theta}{۲} = \frac{\frac{\sqrt{۲}}{۲}}{\frac{\sqrt{۱۳}}{۱}} = \frac{\sqrt{۲}}{۲\sqrt{۱۳}}$

$\sqrt{۱۳}$ | $\sqrt{۲}/۲$ | x → $x = \sqrt{۱۳ - \frac{۱}{۲}} = \sqrt{\frac{۲۵}{۲}}$

(۲)

$\Rightarrow \cos\frac{\theta}{۲} = \frac{\sqrt{\frac{۲۵}{۲}}}{\sqrt{۱۳}}$

$\angle \theta = ۲\angle \frac{\theta}{۲} \cos\frac{\theta}{۲} \rightarrow ۲ \times \frac{\sqrt{۲}}{۲\sqrt{۱۳}} \times \frac{\sqrt{\frac{۲۵}{۲}}}{\sqrt{۱۳}}$

$\Rightarrow \frac{\sqrt{۲۵}}{۱۳} = \boxed{۵/۱۳}$ ✓