تکلیف شماره ۱۲

A

۱)

$$\frac{a\sin\frac{4\pi}{3}+b\cos\frac{7\pi}{6}}{a\sin\frac{11\pi}{6}+b\cos\frac{\pi}{3}}=\tan\frac{5\pi}{6}\Rightarrow\frac{-\frac{a\sqrt3}{2}-\frac{b\sqrt3}{2}}{\frac{-a}{2}+\frac{b}{2}}=\frac{-\sqrt3}{3}\Rightarrow\frac{b+a}{b-a}=\frac{1}{3}\Rightarrow 3b+3a=b-a$$

$$4a=-2b\Rightarrow\frac{b}{a}=-2$$

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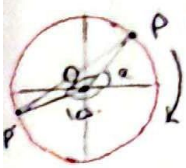
۲)



$$P(\tfrac12,y)\Rightarrow\cos\alpha=\tfrac12\Rightarrow\alpha=60^\circ\Rightarrow\sin\alpha=\tfrac{\sqrt3}{2}\Rightarrow y=\tfrac{\sqrt3}{2}$$

$$210-60=150\Rightarrow P\hat{O}P'=150^\circ$$

قضیه کسینوس‌ها: $PP'^2=OP^2+OP'^2-2\,OP\,OP'\cos(P\hat{O}P')\Rightarrow 1+1-2(1)(1)(-\tfrac{\sqrt3}{2})=2+\sqrt3\Rightarrow PP'=\sqrt{2+\sqrt3}$

$-\frac{\sqrt3}{2}$

$=\sqrt{3+1+1}=2.\sqrt3$

غلط $=2+\sqrt{2+\sqrt3}$ $\quad PP'=\sqrt{2+\sqrt3}$

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۳)

$$\frac{\alpha}{180}=\frac{\frac{\pi}{9}}{\frac{\pi}{1}}\Rightarrow\frac19=\frac{\alpha}{180}\Rightarrow\alpha=20\Rightarrow 20{:}00\longrightarrow 20{:}40$$

٤٠ دقیقه

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۴)

$$\alpha=\left|\frac{11m}{2}-30h\right|\Rightarrow 20=\left|\frac{11m}{2}-90\right|\xrightarrow{1}\frac{11m}{2}-90=20\Rightarrow\frac{11m}{2}=110\Rightarrow m=20$$

$$\xrightarrow{2}90-\frac{11m}{2}=20\Rightarrow\frac{11m}{2}=70\Rightarrow m=\frac{140}{11}$$

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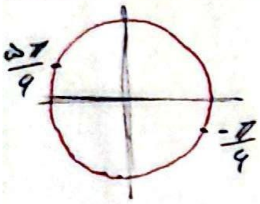
۵)



$$\sin\alpha=\frac{m-3}{5},\ \frac{-\pi}{6}<\alpha<\frac{5\pi}{6}\Rightarrow\frac{-1}{2}<\frac{m-3}{5}\leq1\Rightarrow\frac{-5}{2}<m-3\leq5$$

$$\Rightarrow\frac12<m\leq8$$

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۶)

$$\frac{\sin(\frac{\pi}{2}+\alpha)+k\cos(\frac{3\pi}{2}-\alpha)}{\cos(\pi-\alpha)+2\sin(\alpha+\pi)}=\frac{\cos\alpha-k\sin\alpha}{-\cos\alpha-2\sin\alpha}\xrightarrow{\div\cos\alpha}\frac{1-k\tan\alpha}{-1-2\tan\alpha}=3$$

$\alpha=15$, $\tan\alpha=0.5$

$$\Rightarrow-3-6\tan\alpha=1-k\tan\alpha\Rightarrow1-\frac{k}{4}=\frac{-3}{2}-\frac{6}{4}\Rightarrow\frac{k}{4}=\frac{11}{2}\Rightarrow k=22$$

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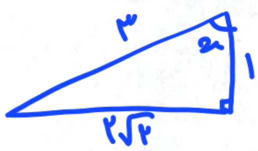
۷)

$$\frac{\tan(\frac{\pi}{2}-x)-\cot(\pi-x)}{\cot(\frac{3\pi}{2}-x)}=\frac{2\cot x}{-\tan x}=\frac{\frac{2\cos x}{\sin x}}{\frac{-\sin x}{\cos x}}=-2\,\frac{\frac{\sqrt3}{2}}{\frac{-1}{2}}=2\sqrt3\Rightarrow\frac{1-\cos x}{1+\cos x}=3$$

$=-2\cot^2 x=-\frac14$

$$\Rightarrow3+3\cos x=1-\cos x\Rightarrow4\cos x=-2\Rightarrow\cos x=\frac{-1}{2}\Rightarrow\sin x=\frac{\sqrt3}{2}$$

$\cot x=-\frac{1}{\sqrt8}$



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۸)

$$\frac{108}{180}=\frac{\alpha}{\pi}\Rightarrow\alpha=\frac{3\pi}{5}\Rightarrow\frac{3\pi}{5}R_1=\frac{9\pi}{10}R_2\Rightarrow\frac{R_1}{R_2}=\frac{\frac{9}{10}}{\frac{3}{5}}=\frac{45}{30}=\frac32$$

$$\Rightarrow\frac{R_1^2}{R_2^2}=\frac94\Rightarrow\frac{\pi R_1^2}{\pi R_2^2}=\frac{9\pi}{4\pi}$$

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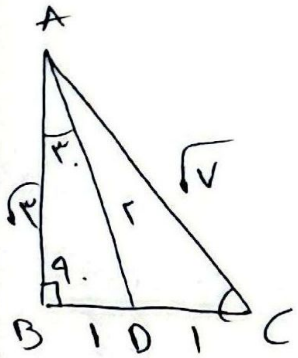

$S_{A\hat{B}C} = \sqrt{3}$

$\sin 30^\circ = \frac{1}{2} = \frac{BD}{AD} \Rightarrow AD = 2$ و $AB = \sqrt{3}$, $\sin \hat{C} = \frac{\sqrt{3}}{\sqrt{7}}$

$\cos \hat{C} = \frac{2}{\sqrt{7}}$

$\sin 2\hat{C} = 2\sin\hat{C}\cos\hat{C} = 2\left(\frac{\sqrt{3}}{\sqrt{7}}\right)\left(\frac{2}{\sqrt{7}}\right) = \frac{4\sqrt{3}}{7}$

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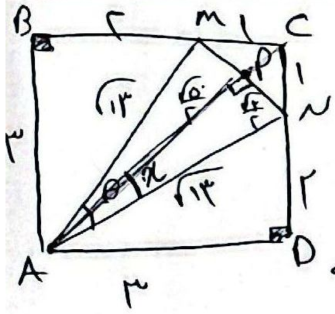

$AN^2 = 9 + 4 = AN^2 = 13 \Rightarrow AN = \sqrt{13}$, $AN = AM \Rightarrow AM = \sqrt{13}$

$MN = \sqrt{2}$ و $AP = \frac{\sqrt{50}}{2}$, $PN = MP = \frac{\sqrt{2}}{2}$

$\sin\alpha = \frac{PN}{AN} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{13}}{1}} = \frac{\sqrt{2}}{2\sqrt{13}}$ و $\cos\alpha = \frac{AP}{AN} = \frac{\frac{\sqrt{50}}{2}}{\frac{\sqrt{13}}{1}} = \frac{\sqrt{50}}{2\sqrt{13}}$

$\sin \underbrace{\theta}_{2\alpha} = 2\sin\alpha\cos\alpha \Rightarrow 2\left(\frac{\sqrt{2}}{2\sqrt{13}}\right)\left(\frac{\sqrt{50}}{2\sqrt{13}}\right) = \frac{\sqrt{100}}{26} = \frac{10}{26} = \frac{5}{13}$

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