ریاضی یازدهم - [illegible]

تکلیف شماره ۱۲

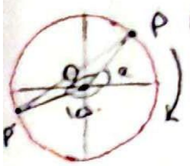
آیار آزادنیا

①

$$\frac{a\sin\frac{۴\pi}{۳}+b\cos\frac{۷\pi}{۶}}{a\sin\frac{۱۱\pi}{۶}+b\cos\frac{\pi}{۳}}=\tan\frac{۵\pi}{۶}\Rightarrow\frac{\frac{-a\sqrt{۳}}{۲}-\frac{b\sqrt{۳}}{۲}}{\frac{-a}{۲}+\frac{b}{۲}}=\frac{-\sqrt{۳}}{۳}\Rightarrow\frac{b+a}{b-a}=\frac{۱}{۳}\Rightarrow ۳b+۳a=b-a$$

$$۴a=-۲b\Rightarrow\frac{b}{a}=-۲$$

②



$$P(\tfrac{۱}{۲},y)\Rightarrow\cos\alpha=\frac{۱}{۲}\Rightarrow\alpha=۶۰^\circ\Rightarrow\sin\alpha=\frac{\sqrt{۳}}{۲}\Rightarrow y=\frac{\sqrt{۳}}{۲}$$

$$۲۱۰-۶۰=۱۵۰\Rightarrow P\hat{O}P'=۱۵۰^\circ$$

قضیه کسینوس‌ها $\Rightarrow PP'^{۲}=OP^{۲}+OP'^{۲}-۲OP\cdot OP'\cos(P\hat{O}P')\Rightarrow ۱+۱-۲(۱)(۱)(-\frac{۱}{۲})=۳\Rightarrow PP'=\sqrt{۳}$

$$\Rightarrow\sqrt{۳+۱+۱}=۲+\sqrt{۳}$$

③

$$\frac{\alpha}{۱۸۰}=\frac{\frac{\pi}{۹}}{\frac{\pi}{۱}}\Rightarrow\frac{۱}{۹}=\frac{\alpha}{۱۸۰}\Rightarrow\alpha=۲۰\Rightarrow ۲:۰۰\longrightarrow ۲:٤۰$$

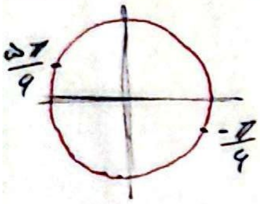
٤۰ دقیقه

④

$$\alpha=\left|\frac{۱۱m}{۲}-۳۰h\right|\Rightarrow ۲۰=\left|\frac{۱۱m}{۲}-۹۰\right|\xrightarrow{①}\frac{۱۱m}{۲}-۹۰=۲۰\Rightarrow\frac{۱۱m}{۲}=۱۱۰\Rightarrow m=۲۰\ ✓$$

$$\xrightarrow{②} ۹۰-\frac{۱۱m}{۲}=۲۰\Rightarrow\frac{۱۱m}{۲}=۷۰\Rightarrow m=\frac{۱۴۰}{۱۱}$$

⑤



$$\sin\alpha=\frac{m-۳}{۵}\ ,\ \frac{-\pi}{۶}<\alpha<\frac{۵\pi}{۶}\Rightarrow\frac{-۱}{۲}<\frac{m-۳}{۵}\leq ۱\Rightarrow\frac{-۵}{۲}<\ -۳\leq ۵$$

$$\Rightarrow\frac{۱}{۲}<m\leq ۸$$

⑥

$$\frac{\sin(\frac{\pi}{۲}+\alpha)+K\cos(\frac{۳\pi}{۲}-\alpha)}{\cos(\pi-\alpha)+۲\sin(\alpha+\pi)}=\frac{\cos\alpha-K\sin\alpha}{-\cos\alpha-۲\sin\alpha}\xrightarrow{\div\cos\alpha}\frac{۱-K\tan\alpha}{-۱-۲\tan\alpha}=۳$$

$\alpha=۱۵$
$\tan\alpha=۰/۵$

$$\Rightarrow -۳-۶\tan\alpha=۱-K\tan\alpha\Rightarrow ۱-\frac{K}{۴}=\frac{-۳}{۲}-\frac{۶}{۲}\Rightarrow\frac{K}{٤}=\frac{۱۱}{۲}\Rightarrow K=۲۲$$

⑦

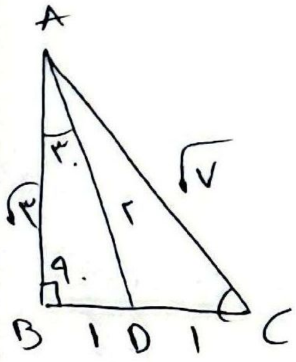
$$\frac{\tan(\frac{\pi}{۲}-x)-\cot(\pi-\alpha)}{\cot(\frac{۳\pi}{۲}-\alpha)}=\frac{۲\cot x}{-\tan\alpha}=\frac{\frac{۲\cos x}{\sin x}}{\frac{-\sin x}{\cos x}}=-۲\,\frac{\frac{\sqrt{۳}}{۲}}{\frac{-۱}{۲}}=۲\sqrt{۳}\Rightarrow\frac{۱-\cos x}{۱+\cos x}=۳$$

$$\Rightarrow ۳+۳\cos x=۱-\cos x\Rightarrow ۴\cos x=-۲\Rightarrow\cos x=\frac{-۱}{۲}\Rightarrow\sin x=\frac{\sqrt{۳}}{۲}$$

⑧

$$\frac{۱۰۸}{۱۸۰}=\frac{\alpha}{\pi}\Rightarrow\alpha=\frac{۳\pi}{۵}\Rightarrow\frac{۳\cancel{\pi}}{۵}R_۱=\frac{۹\cancel{\pi}}{۱۰}R_۲\Rightarrow\frac{R_۱}{R_۲}=\frac{\frac{۹}{۱۰}}{\frac{۳}{۵}}=\frac{٤۵}{۳۰}=\frac{۳}{۲}$$

$$\Rightarrow\frac{R_۱^{۲}}{R_۲^{۲}}=\frac{۹}{٤}\Rightarrow\frac{\pi R_۱^{۲}}{\pi R_۲^{۲}}=\frac{۹\cancel{\pi}}{٤\cancel{\pi}}$$



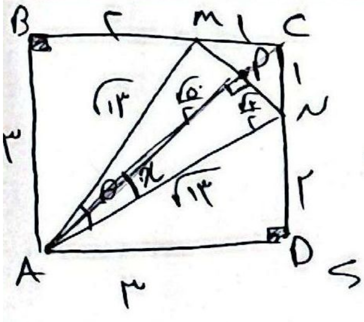
⑨

$S_{A\hat{B}C} = \sqrt{۳}$

$\sin ۳۰^{\circ} = \frac{۱}{۲} = \frac{BD}{AD} \Rightarrow AD = ۲$ و $AB = \sqrt{۳}$, $\sin \hat{C} = \frac{\sqrt{۳}}{\sqrt{۷}}$

$\cos \hat{C} = \frac{۲}{\sqrt{۷}}$

$\sin ۲\hat{C} = ۲\sin \hat{C} \cos \hat{C} = ۲\left(\frac{\sqrt{۳}}{\sqrt{۷}}\right)\left(\frac{۲}{\sqrt{۷}}\right) = \frac{۴\sqrt{۳}}{۷}$



⑩

$AN^۲ = ۹+۴ = AN^۲ = ۱۳ \Rightarrow AN = \sqrt{۱۳}$, $AN = AM \Rightarrow AM = \sqrt{۱۳}$

$MN = \sqrt{۲}$ و $AP = \frac{\sqrt{۵۰}}{۲}$, $PN = MP = \frac{\sqrt{۲}}{۲}$

$\sin \alpha = \frac{PN}{AN} = \frac{\frac{\sqrt{۲}}{۲}}{\frac{\sqrt{۱۳}}{۱}} = \frac{\sqrt{۲}}{۲\sqrt{۱۳}}$ و $\cos \alpha = \frac{AP}{AN} = \frac{\frac{\sqrt{۵۰}}{۲}}{\frac{\sqrt{۱۳}}{۱}} = \frac{\sqrt{۵۰}}{۲\sqrt{۱۳}}$

$\sin \underbrace{\theta}_{۲\alpha} = ۲\sin \alpha \cos \alpha \Rightarrow \cancel{۲}\left(\frac{\sqrt{۲}}{\cancel{۲}\sqrt{۱۳}}\right)\left(\frac{\sqrt{۵۰}}{۲\sqrt{۱۳}}\right) = \frac{\sqrt{۱۰۰}}{۲۶} = \frac{۱۰}{۲۶} = \frac{۵}{۱۳}$