Subject: Date:

Mo Tu We Th Fr Sa Su

شماره ۶ ۱۲ شنتا پیرای

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$$\frac{a\sin(\frac{4\pi}{3})+b\cos(\frac{7\pi}{6})}{a\sin(\frac{11\pi}{6})+b\cos\frac{\pi}{3}}=\tan\frac{5\pi}{6} \Rightarrow \frac{-\frac{\sqrt{3}}{2}a-\frac{\sqrt{3}}{2}b}{-\frac{1}{2}a+\frac{1}{2}b}=-\frac{\sqrt{3}}{3} \Rightarrow -\sqrt{3}\left(\frac{a+b}{b-a}\right)=-\frac{\sqrt{3}}{3}$$

$$\Rightarrow \frac{a+b}{b-a}=\frac{1}{3} \Rightarrow 3a+3b=b-a \Rightarrow 4a=-2b \Rightarrow -2a=b \Rightarrow \boxed{\frac{b}{a}=-2}$$

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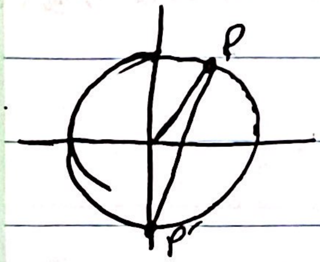

$$\cos\alpha=\frac{1}{2} \xrightarrow{0<\alpha<90} \alpha=60^\circ \Rightarrow \alpha+210=270^\circ \Rightarrow P'(0,-1)$$

$$\sin\alpha=\sin 60^\circ=\frac{\sqrt{3}}{2} \Rightarrow P\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$$

$$PP'=\sqrt{(x_P-x_{P'})^2+(y_P-y_{P'})^2}=\sqrt{\left(\frac{1}{2}-0\right)^2+\left(\frac{\sqrt{3}}{2}-(-1)\right)^2}$$

$$=\sqrt{\frac{1}{4}+\left(\frac{3}{4}+1+\sqrt{3}\right)}=\sqrt{2+\sqrt{3}}=\sqrt{\frac{1}{2}(4+2\sqrt{3})}=\sqrt{\frac{1}{2}(1+\sqrt{3})^2}$$

$$\Rightarrow PP'=\frac{1+\sqrt{3}}{\sqrt{2}}=\frac{\sqrt{2}+\sqrt{6}}{2} \Rightarrow \text{محیط } O\hat{P}P'=(1+1)+\left(\frac{\sqrt{2}+\sqrt{6}}{2}\right)=\boxed{2+\frac{\sqrt{2}+\sqrt{6}}{2}}$$

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ابتدا رادیان را به درجه تبدیل می‌کنیم

$$\frac{\frac{\pi}{9}}{\pi}=\frac{Degree}{180} \Rightarrow Deg=20^\circ$$

می‌دانیم در ساعت H:M، عقربه ساعت‌شمار $\frac{M}{2}$ درجه از شماره ساعت خود جلوتر آمده است (زیرا در هر دقیقه $\frac{1}{2}$ درجه می‌رود)

لذا در این ساعت $2:M'$ داریم:

$$\frac{M}{2}=20 \Rightarrow M=40' \Rightarrow \boxed{\text{ساعت}: 2:40'}$$

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α زاویه بین دو عقربه ساعت و دقیقه‌شمار $=\left|\frac{11}{2}M-30H\right| \Rightarrow 20^\circ=\left|\frac{11}{2}M-30(3)\right|=\left|\frac{11M}{2}-90\right|$

$$\Rightarrow \begin{cases} 20=\frac{11}{2}M-90 \Rightarrow \frac{11}{2}M=110 \Rightarrow \boxed{M=20'} \rightarrow \text{بار دوم } \checkmark \\ \frac{11}{2}M-90=-20 \Rightarrow \frac{11}{2}M=70 \Rightarrow M=\frac{140}{11}=12/\text{خورده} \rightarrow \text{بار اول } \times \end{cases}$$

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$\Rightarrow -\frac{\pi}{6} \leq x \leq \frac{5\pi}{6} \Rightarrow -\frac{1}{2} \leq \sin x \leq 1 \Rightarrow -\frac{1}{2} \leq \frac{m-3}{5} \leq 1 \Rightarrow -\frac{5}{2} \leq m-3 \leq 5$

$\Rightarrow \frac{1}{2} \leq m \leq 8 \xrightarrow[\text{(اعداد صحیح)}]{} m = \{1,2,3,4,5,6,7,8\} \rightarrow$ $\boxed{\text{۸ عدد صحیح}}$

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$\tan 15^\circ = \frac{1}{4} \Rightarrow \frac{\sin 15^\circ}{\cos 15^\circ} = \frac{1}{4} \Rightarrow \cos 15^\circ = 4\sin 15^\circ$

$= \frac{\sin(90^\circ+15^\circ) + k\cos(270^\circ-15^\circ)}{\cos(180^\circ-15^\circ) + 2\sin(180^\circ+15^\circ)} = \frac{\cos 15^\circ - k\sin 15^\circ}{-\cos 15^\circ - 2\sin 15^\circ} = \frac{4\sin 15^\circ - k\sin 15^\circ}{-4\sin 15^\circ - 2\sin 15^\circ}$

$= \frac{(4-k)\sin 15^\circ}{-6\sin 15^\circ} = \frac{k-4}{6} = 3 \Rightarrow k-4 = 18 \Rightarrow \boxed{k=22}$

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$\frac{1-\cos x}{1+\cos x} = 2 \Rightarrow 1-\cos x = 2+2\cos x \Rightarrow 3\cos x = -1 \Rightarrow \cos x = -\frac{1}{3} \Rightarrow \cos^2 x = \frac{1}{9}$

$\Rightarrow \frac{1}{\cos^2 x} = 9 = 1+\tan^2 x \Rightarrow \tan^2 x = 8 \Rightarrow \cot^2 x = \frac{1}{8}$

$\frac{\tan(\frac{\pi}{2}-x) - \cot(\pi-x)}{\cot(\frac{3\pi}{2}-x) + 2\tan(2\pi-x)} = \frac{\cot x - (-\cot x)}{\tan x - 2\tan x} = \frac{2\cot x}{-\tan x} = -2\cot^2 x = -2\left(\frac{1}{8}\right) = -\frac{1}{4}$

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$\frac{108}{180} = \frac{Rad}{\pi} \Rightarrow Rad = \frac{3\pi}{5}$ ابتدا زاویه ۱۰۸° را به رادیان تبدیل می‌کنیم

$l = R\theta$

طول کمان دایره A $= R_A \times \frac{3\pi}{5}$

طول کمان دایره B $= R_B \times \frac{9\pi}{10}$

طول کمان‌ها برابر $\Rightarrow R_A \times \frac{3\pi}{5} = R_B \times \frac{9\pi}{10} \Rightarrow \frac{R_A}{R_B} = \frac{3}{2} \Rightarrow \frac{R_A^2}{R_B^2} = \frac{9}{4} = \frac{\pi R_A^2}{\pi R_B^2}$

$\Rightarrow \frac{S_A}{S_B} = \frac{9}{4}$

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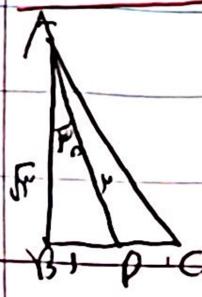

$\sin 30^\circ = \frac{BD}{AD} \Rightarrow \frac{1}{AD} = \frac{1}{2} \Rightarrow AD = 2, \quad \cos 30^\circ = \frac{AB}{AD} \Rightarrow \frac{AB}{2} = \frac{\sqrt{3}}{2} \Rightarrow AB = \sqrt{3}$

$S_{\hat{ABC}} = \frac{1}{2} \times \sqrt{3} \times BC = \sqrt{3} \Rightarrow BC = 2 \xrightarrow{\text{فیثاغورس}} AC^2 = (2)^2 + (\sqrt{3})^2 = 7 \Rightarrow AC = \sqrt{7}$

$\Rightarrow \sin \hat{C} = \frac{AB}{AC} = \frac{\sqrt{3}}{\sqrt{7}}, \quad \cos \hat{C} = \frac{BC}{AC} = \frac{2}{\sqrt{7}} \Rightarrow \sin 2\hat{C} = 2\sin\hat{C}\cos\hat{C}$

$\Rightarrow \sin 2\hat{C} = 2\left(\frac{\sqrt{3}}{\sqrt{7}}\right)\left(\frac{2}{\sqrt{7}}\right) = \boxed{\frac{4\sqrt{3}}{7}}$

۱۰

$$EH \perp AD,\ CD \perp AD \Rightarrow EH \parallel CD \xRightarrow{\text{قضیه تالس}} \frac{EH}{ND} = \frac{AH}{AD} \Rightarrow \frac{EH}{۲} = \frac{۲}{۳} \Rightarrow EH = \frac{۴}{۳}$$

$$\tan\alpha = \frac{\frac{۴}{۳}}{۲} = \frac{۲}{۳} \quad , \quad \tan(\alpha+\theta) = \frac{۳}{۲}$$ (بنا به تعریف تانژانت)

$$\tan(\alpha+\theta) = \frac{\tan\alpha + \tan\theta}{1 - \tan\alpha\tan\theta} \Rightarrow \frac{\frac{۲}{۳} + \tan\theta}{1 - \frac{۲}{۳}\tan\theta} = \frac{۳}{۲} \Rightarrow \frac{۲}{۳} + \tan\theta = \frac{۳}{۲} - \tan\theta$$

$$\Rightarrow ۲\tan\theta = \frac{۳}{۲} - \frac{۲}{۳} = \frac{۵}{۶} \Rightarrow \tan\theta = \frac{۵}{۱۲} \Rightarrow \cot\theta = \frac{۱۲}{۵} \Rightarrow 1 + \cot^2\theta = \frac{1}{\sin^2\theta}$$

$$\Rightarrow 1 + \frac{۱۴۴}{۲۵} = \frac{۱۶۹}{۲۵} = \frac{1}{\sin^2\theta} \Rightarrow \sin^2\theta = \frac{۲۵}{۱۶۹} \xRightarrow[\text{کوچکتر از زاویه مربع } (۹۰^\circ)]{0 < \theta < ۹۰} \boxed{\sin\theta = +\frac{۵}{۱۳}}$$