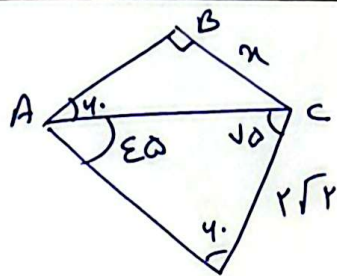


$$\frac{a}{\sin A} = \frac{b}{\sin B} \rightarrow \frac{2}{\frac{\sqrt{2}}{2}} = \frac{2\sqrt{2}}{\sin B} \rightarrow \sin B = \frac{1}{\sqrt{2}}$$

$B = 45^\circ$

$c = \sqrt{a^2 + b^2} = \sqrt{4 + 8} = \sqrt{12} = 2\sqrt{3}$



$$\frac{2\sqrt{2}}{\sin 40} = \frac{AC}{\sin 50} \rightarrow \frac{2\sqrt{2}}{\frac{1}{\sqrt{2}}} = \frac{AC}{\frac{\sqrt{2}}{2}} \rightarrow AC = 2\sqrt{2}$$

$x = AC \sin 40 = 2\sqrt{2} \times \frac{1}{\sqrt{2}} = 2$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$$a = k \sin A \Rightarrow (b^2 - r^2) \frac{\sin C}{\sin B} = k \sin C \rightarrow b^2 - r^2 = k \sin B$$

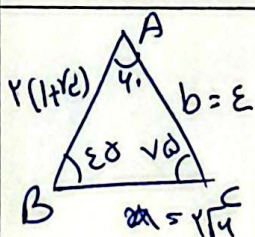
$$b = k \sin B \rightarrow b^2 - b^2 - r^2 = 0 \Rightarrow b^2 - b - r^2 = 0$$

$b = 2$

$$(13)^2 = (2)^2 + (21)^2 - 2(2)(21) \cos B \rightarrow 169 = 4 + 441 - 84 \cos B \rightarrow 84 \cos B = 276 \rightarrow \cos B = \frac{276}{84} = \frac{23}{7}$$

$$\rightarrow \cos B = \frac{23}{7} = 0.18 = \frac{2}{11}$$

$$\sin^2 B + \cos^2 B = 1 \rightarrow \sin^2 B + \frac{4}{121} = 1 \rightarrow \sin^2 B = \frac{117}{121} \rightarrow \sin B = \frac{9}{11}$$



$$a^2 = 14 + 4(1 + 2 + 2\sqrt{2}) - (2)(4)(2 + 2\sqrt{2})\left(\frac{1}{\sqrt{2}}\right)$$

$$a^2 = 24 \rightarrow a = \sqrt{24} = 2\sqrt{6}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} \rightarrow \frac{2\sqrt{6}}{\frac{\sqrt{2}}{2}} = \frac{4}{\sin B} \rightarrow \sin B = \frac{1}{\sqrt{2}} \rightarrow B = 45^\circ$$

$c = \sqrt{a^2 + b^2} = \sqrt{24 + 8} = \sqrt{32} = 4\sqrt{2}$

در مثلث ABC : $x = 1 + x - (2)(1)(4) \cos \alpha \rightarrow x \cos \alpha = 1 \rightarrow \cos \alpha = \frac{1}{4}$ ✓

در مثلث DEF : $DE = 14 + 4 - (2)(4)(4) \cos \alpha \Rightarrow DE = 10$

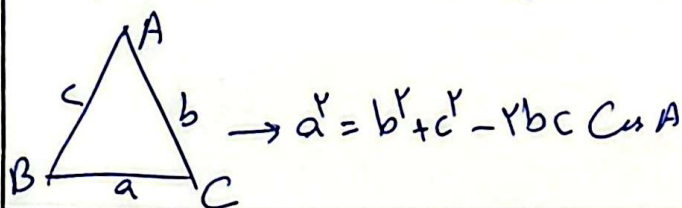
(2)

$DE = 10$ ✓ جواب

الف) قانون کینوس ها
 $BC^2 = 2^2 + 4^2 - (2)(2)(1) \cos 45^\circ = 10$
 $BC = \sqrt{10}$ ✓ جواب

ب) $S = \frac{1}{2} ab \sin \alpha \rightarrow \frac{1}{2} \times 2 \times 1 \times \frac{\sqrt{10}}{4} = \frac{\sqrt{10}}{4}$ ✓ جواب (2)

$\frac{(b+c)^2 - a^2}{bc(\cos A + 1)} = \frac{b^2 + c^2 + 2bc - a^2}{bc(\cos A + 1)} = \frac{2bc(\cos A + 1)}{bc(\cos A + 1)} = 2$ ✓ جواب (2)



$\frac{a^2 + b^2 - c^2}{a + b - c} = a^2 \rightarrow a^2 + b^2 - c^2 = a^2 + a^2 b - a^2 c$
 $\rightarrow (b - c)(b^2 + c^2 + bc) = a^2(b - c)$ ✓ جواب
 $\rightarrow b^2 + c^2 + bc = b^2 + c^2 - 2bc \cos A \rightarrow bc = -2bc \cos A$
 $\cos A = -\frac{1}{2} \rightarrow A = 120^\circ$ ✓ جواب (2)

$S = \frac{1}{2} ab \sin \alpha \rightarrow 2 = \frac{1}{2} (3 - \cos \beta)(1 + 4 \cos \beta) \times \frac{\sqrt{10}}{4}$
 $\rightarrow 3 + 4 \cos \beta - \cos \beta - 3 \cos^2 \beta = 1 \rightarrow 3 \cos^2 \beta - 1 \cos \beta + 2 = 0$
 $\rightarrow \cos \beta = \frac{1}{3} \rightarrow \beta = \arccos \frac{1}{3}$
 $a^2 = 4 + 14 - (2)(7)(\frac{1}{3}) \times \frac{\sqrt{10}}{4} = 10 - \frac{7\sqrt{10}}{2}$ ✓ جواب (2)

