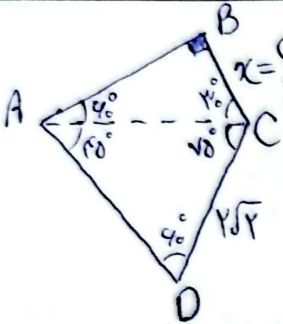


$$\frac{a}{\sin A} = \frac{b}{\sin B} \rightarrow \frac{20}{\sin 60^\circ} = \frac{10\sqrt{6}}{3 \sin B} \rightarrow \frac{20}{\sqrt{3}} = \frac{10\sqrt{6}}{3 \sin B} \rightarrow \frac{2}{\sqrt{3}} = \frac{\sqrt{6}}{3 \sin B}$$

$$\rightarrow 6 \sin B = 3\sqrt{2} \rightarrow \sin B = \frac{\sqrt{2}}{2} \rightarrow \hat{B} = 45^\circ$$

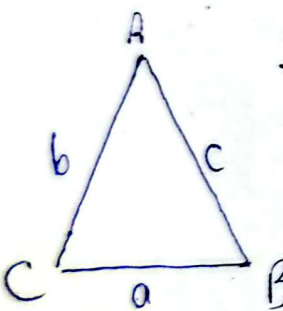
$$\rightarrow \hat{B} + \hat{C} = 45^\circ + \hat{C} = 110^\circ \rightarrow \hat{C} = 65^\circ$$



$$\frac{a}{\sin A} = \frac{d}{\sin D} \rightarrow \frac{20}{\sin 60^\circ} = \frac{d}{\sin 90^\circ} \rightarrow d = AC = 2\sqrt{3}$$

نقطه مقابل زاویه ۶۰ در مثل ABC برابر وتر است

$$\overline{BC} = x = \frac{\sqrt{3}}{2} \overline{AC} = \frac{\sqrt{3}}{2} \times 2\sqrt{3} = 3$$

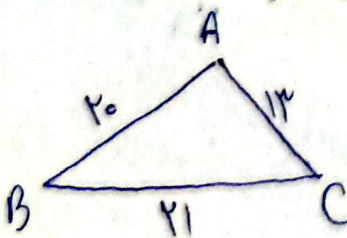


$$(b^2 - 2) \frac{\sin C}{\sin B} = c \xrightarrow{\div \sin C} \frac{c}{\sin C} = \frac{b}{\sin B} = \frac{b^2 - 2}{\sin B}$$

$$\rightarrow b = b^2 - 2 \rightarrow b^2 - b - 2 = 0 \rightarrow b = -1 \times \text{و } b = 2$$

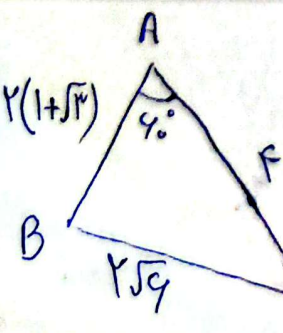
با توجه به شکل B؛ زاویه ای مثل Sin و Cos مثبت دارد

$$b^2 = a^2 + c^2 - 2ac \cos B \rightarrow 149 = 121 + 120 \cos B$$



$$\rightarrow -672 = -120 \cos B \rightarrow \cos B = \frac{2}{5}$$

$$\rightarrow \cos^2 B + \sin^2 B = 1 \rightarrow \sin^2 B = 1 - \frac{4}{25} = \frac{21}{25} \rightarrow \sin B = \frac{\sqrt{21}}{5}$$

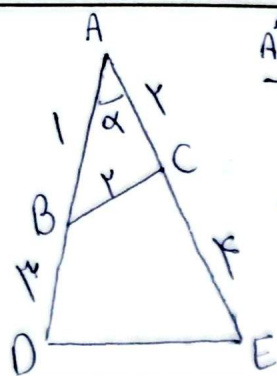


الف) $a^2 = b^2 + c^2 - 2bc \cos A \rightarrow a^2 = 16 + 12 + 12\sqrt{3} - 12\sqrt{3} = 28 \rightarrow a = 2\sqrt{7}$

$$\frac{a}{\sin A} = \frac{b}{\sin B} \rightarrow \frac{2\sqrt{7}}{\sin 60^\circ} = \frac{2}{\sin B} \rightarrow 2\sqrt{7} = \frac{2}{\sin B} \rightarrow \sin B = \frac{1}{\sqrt{7}}$$

$$\rightarrow \sin B = \frac{\sqrt{7}}{7} \rightarrow \hat{B} = 45^\circ$$

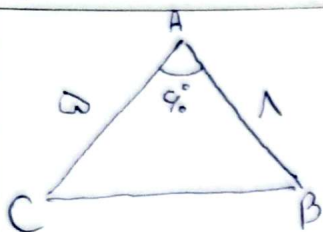
$$\rightarrow \hat{A} + \hat{B} + \hat{C} = 180^\circ \rightarrow \hat{C} = 70^\circ$$



$$\triangle ABC \rightarrow a^2 = b^2 + c^2 - 2bc \cos \alpha \rightarrow \xi = \omega - \xi \cos \alpha \rightarrow \cos \alpha = \frac{1}{\xi}$$

$$\triangle ADE \rightarrow a^2 = d^2 + e^2 - 2de \cos \alpha \rightarrow a^2 = y^2 + l^2 - 2yl \left(\cos \alpha \right) = \frac{1}{\xi}$$

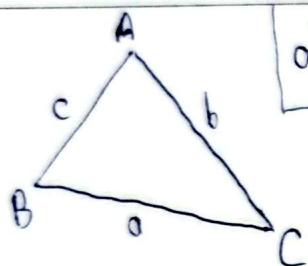
$$\rightarrow a^2 = 2y - 1 = \xi_0 \rightarrow a = \sqrt{\xi_0} = \sqrt{10}$$



$$a^2 = b^2 + c^2 - 2bc \cos 90^\circ = 10 + 4 - 1 \cdot 0 \left(\frac{1}{\sqrt{2}} \right) = 14 \quad (\text{الف})$$

$$\rightarrow a = \sqrt{14}$$

$$S_{\triangle ABC} = \frac{1}{2} bc \sin 90^\circ = \frac{1}{2} (1)(1) \left(\frac{\sqrt{2}}{2} \right) = 10\sqrt{2}$$



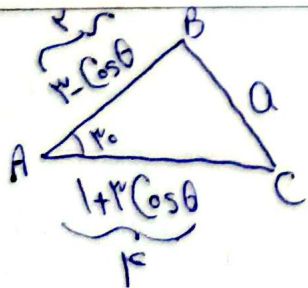
$$a^2 = b^2 + c^2 - 2bc \cos \hat{A} \Rightarrow \frac{(b+c)^2 - a^2}{bc(\cos \hat{A} + 1)} = \frac{b^2 + c^2 + 2bc - (b^2 + c^2 - 2bc \cos \hat{A})}{bc(\cos \hat{A} + 1)}$$

$$= \frac{2bc + 2bc \cos \hat{A}}{bc(\cos \hat{A} + 1)} = \frac{2bc(\cos \hat{A} + 1)}{bc(\cos \hat{A} + 1)} = 2$$

$$\rightarrow a^2 + b^2 - c^2 = a^2 + ab - ac \rightarrow b^2 + c^2 = a^2 b + a^2 c \rightarrow (b+c)(b^2 + c^2) = a^2(b+c)$$

$$\rightarrow \frac{b^2 + c^2 + abc}{b^2 + c^2 - 2bc \cos \hat{A}} = a^2 \rightarrow bc = -2bc \cos \hat{A} \rightarrow \cos \hat{A} = -\frac{1}{2}$$

$$\left(\frac{1}{2} \right) \Rightarrow \hat{A} = 120^\circ$$



$$\rightarrow S_{\triangle ABC} = \frac{1}{2} (r - \cos \theta)(1 + r \cos \theta) \frac{1}{r} = r$$

$$\rightarrow r + r \cos \theta - \cos \theta - r \cos^2 \theta = 1 \rightarrow r \cos \theta - 1 \cos \theta + 1 = 0$$

$$\cos \theta = 1$$

$$\cos \theta = \frac{\omega}{r} \cos \theta$$

$$\Rightarrow a^2 = b^2 + c^2 - 2bc \cos 120^\circ = 14 + 4 - 1 \cdot 0 \left(\frac{\sqrt{2}}{2} \right) = 10 - 1\sqrt{2}$$