


 $\Rightarrow \hat{A} = 110 - 120 = 90 \Rightarrow \frac{a}{\sin A} = \frac{b}{\sin B}$
 $\Rightarrow \frac{14}{\sin 40} = \frac{12}{\sin B} \Rightarrow \sin B = \frac{12 \sin 40}{14} \Rightarrow \sin B = \frac{6 \sin 40}{7} \Rightarrow \hat{B} = 50$
 $\hat{C} = 110 - 50 - 40 = 20$

$\hat{A}_1 = 40 \Rightarrow \hat{A}_2 = 110 - 40 - 20 = 50 \Rightarrow DC = \frac{1}{\sqrt{2}} AC \Rightarrow (AC \text{ وتر})$
 $\Rightarrow \frac{AC}{\sin D} = \frac{DC}{\sin A} \Rightarrow \frac{AC}{\frac{1}{\sqrt{2}}} = \frac{DC}{\frac{1}{\sqrt{2}}} \Rightarrow \frac{AC}{\sqrt{2}} = \frac{DC}{\sqrt{2}} \Rightarrow AC = DC$
 $x = \frac{1}{\sqrt{2}} AC = \frac{1}{\sqrt{2}} \times \sqrt{2} = 1$

$\frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{\sin C}{\sin B} = \frac{c}{b} \Rightarrow (b^2 - r^2) \alpha \frac{\sin C}{\sin B} = c$
 $\Rightarrow (b^2 - r^2) \frac{c}{b} = c \Rightarrow \frac{b^2 - r^2}{b} = 1 \Rightarrow b^2 - b - r = 0 \Rightarrow b = 2$

$b^2 = a^2 + c^2 - 2ac \cos B \Rightarrow 149 = 100 + 49 - 140 \cos B$
 $\Rightarrow 140 \cos B = 9 \Rightarrow \cos B = \frac{9}{140} \Rightarrow 1 - \cos^2 B = \sin^2 B$
 $\sin B = \sqrt{1 - \frac{81}{19600}} = \sqrt{\frac{19519}{19600}} = \frac{\sqrt{19519}}{140}$

$a^2 = 14^2 + 14^2 + 14\sqrt{2} = (14^2 + 14^2 + 14\sqrt{2}) \times \frac{1}{2} \Rightarrow a = \sqrt{14^2 + 14^2 + 14\sqrt{2}} = \sqrt{14^2 + 14\sqrt{2}}$

$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{14\sqrt{2}}{\frac{1}{\sqrt{2}}} = \frac{14}{\sin B} \Rightarrow 14\sqrt{2} = \frac{14}{\sin B} \Rightarrow \sin B = \frac{1}{\sqrt{2}}$
 $\Rightarrow \hat{B} = 45$

$$BC^2 = AB^2 + AC^2 - 2AB \cdot AC \cdot \cos \alpha \Rightarrow 5 = 4 + 4 - 2 \cdot 2 \cdot 2 \cdot \cos \alpha \Rightarrow \cos \alpha = \frac{1}{2}$$

$$\Rightarrow DE^2 = AD^2 + AE^2 - 2AD \cdot AE \cdot \cos \alpha \Rightarrow DE^2 = 1^2 + 3^2 - 1 \cdot 3 = 7$$

$$DE = \sqrt{7} = \sqrt{7} \text{ cm}$$

$$a = \sqrt{b^2 + c^2 - 2bc \cos A} = \sqrt{2^2 + 4^2 - 2 \cdot 2 \cdot 4 \cdot \frac{1}{2}} = \sqrt{12} = 2\sqrt{3} \quad (2)$$

$$S_{ABC} = \frac{1}{2} b \cdot c \cdot \sin \theta = \frac{1}{2} \cdot 2 \cdot 4 \cdot \frac{\sqrt{3}}{2} = 2\sqrt{3} \quad (3)$$

$$* a^2 = b^2 + c^2 - 2bc \cos A \Rightarrow b^2 + c^2 + 2bc - a^2 = b^2 + c^2 + 2bc - b^2 - c^2 + 2bc \cos A$$

$$\Rightarrow \frac{2bc + 2bc \cos A}{b(c \cos A + 1)} = \frac{2bc(\cos A + 1)}{b(c \cos A + 1)} = 2$$

$$\frac{a^2 + (b-c)(b^2 + c^2 + bc)}{b + a - c} \Leftrightarrow \frac{a^2(b-a-c)}{(b+a-c)} \Leftrightarrow a^2$$

اثبات با روشی دیگر

پس فقط زمانی برقرار است که $b^2 + c^2 + bc$ برابر با a^2 باشد پس

$$a^2 = b^2 + c^2 + bc \Rightarrow \cos A = -\frac{1}{2}$$

$$S = \frac{1}{2} b \cdot c \cdot \frac{1}{2} = 2 \Rightarrow bc = 1 = (3-t)(1+3t) = 3 - 3t + 1t + 3t^2$$

$$\Rightarrow -3t^2 + 1t - 2 = 0 \Rightarrow \cos \theta = \frac{1}{2} \checkmark \Rightarrow \cos \theta = \frac{1}{2} \times \Rightarrow \begin{matrix} AB = 3 \\ AC = 4 \end{matrix}$$

$$a^2 = 4 + 16 - (2 \cdot 4 \cdot \frac{1}{2}) = 12 = 2 \cdot 2 \cdot 3$$