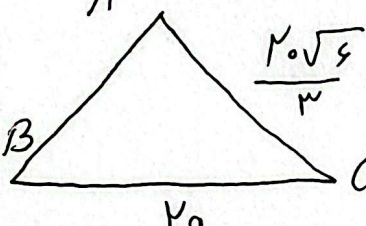


$B+C=120^\circ \Rightarrow A=60^\circ$



$$\frac{BC}{\sin A} = \frac{AC}{\sin B} \Rightarrow \frac{10}{\frac{\sqrt{3}}{2}} = \frac{10\sqrt{3}}{\sin B}$$

$$\Rightarrow \sin B = \frac{\sqrt{18}}{5} = \frac{3\sqrt{2}}{5} = \frac{\sqrt{2}}{2} \Rightarrow B=45^\circ$$

$A=180-(90+75)=15^\circ$

$$\frac{CD}{\sin A} = \frac{AC}{\sin 45^\circ} \Rightarrow \frac{2\sqrt{2}}{\sin 15^\circ} = \frac{AC}{\sin 45^\circ}$$

$$\Rightarrow \frac{2\sqrt{2}}{\frac{\sqrt{2}}{2}} = \frac{AC}{\frac{\sqrt{2}}{2}} \Rightarrow \frac{2AC}{\sqrt{2}} = 2 \Rightarrow AC=2\sqrt{2}$$

$$\frac{x}{\sin 45^\circ} = \frac{2\sqrt{2}}{\sin 90^\circ} \Rightarrow \frac{x}{\frac{\sqrt{2}}{2}} = \frac{2\sqrt{2}}{1} \Rightarrow x=2$$

$$\frac{c}{\sin C} = \frac{b}{\sin B} = \frac{a}{\sin A} = k \Rightarrow c = k \sin C \Rightarrow (b \neq 2) \frac{\sin C}{\sin B} = k \sin C$$

$$\Rightarrow \frac{b^2 - 2}{\sin B} = k \Rightarrow b^2 - 2 = k \sin B \Rightarrow b^2 - b - 2 = 0 \Rightarrow (b-2)(b+1) = 0$$

$$\Rightarrow \boxed{b=2 \checkmark}$$

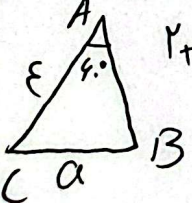
$b=-1 \times$

$$AC^2 = BC^2 + AB^2 - 2BC \cdot AB \cdot \cos B$$

$$149 = 441 + 400 - 2(42 \cdot \cos B) \Rightarrow -672 = -84 \cos B$$

$$\Rightarrow \cos B = 8 \Rightarrow \frac{44}{100} + \sin^2 B = 1 \Rightarrow \sin^2 B = \frac{56}{100} \Rightarrow \sin B = \pm \frac{\sqrt{14}}{5}$$

Case 1 $B < 90^\circ$ $\Rightarrow \sin B = \frac{\sqrt{14}}{5}$



الف) $a^2 = b^2 + c^2 - 2bc \cos A = 14 + 4 + 11\sqrt{3} + 12 - 2(4)(10\sqrt{3}) \frac{1}{2}$

$$\Rightarrow 14 + 14 + 11\sqrt{3} - 1 - 11\sqrt{3} \Rightarrow 24 = a^2 \Rightarrow a = \sqrt{24} = 2\sqrt{6}$$

ب) $\frac{a}{\sin 60^\circ} = \frac{c}{\sin B} \Rightarrow \frac{2\sqrt{6}}{\frac{\sqrt{3}}{2}} = \frac{4}{\sin B} \Rightarrow \sin B = \frac{1}{\sqrt{2}}$

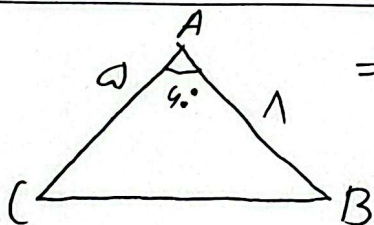
$$\Rightarrow \sin B = \frac{\sqrt{2}}{2} \Rightarrow B = 45^\circ$$

$$BC^2 = AC^2 + AB^2 - 2AC \times AB \times \cos \alpha \Rightarrow E = E + 1 - 2(E \times 1 \times \cos \alpha)$$

$$\Rightarrow -1 = -2 \cos \alpha \Rightarrow \cos \alpha = \frac{1}{2}$$

$$DE^2 = AE^2 + AD^2 - 2AE \times AD \times \cos \alpha = 9 + 1 - 2(3 \times 1 \times \frac{1}{2})$$

$$\Rightarrow DE^2 = 9 - 1 = 8 \Rightarrow DE = \sqrt{8} = 2\sqrt{2}$$



$$\Rightarrow BC^2 = (1)^2 + (2)^2 - 2 \times 1 \times 2 \times \cos \alpha$$

$$\Rightarrow BC^2 = 9 + 4 - 4 \Rightarrow BC^2 = 9 \Rightarrow BC = 3$$

$$S = \frac{1}{2} \times 1 \times 2 \times \sin \alpha = \frac{E \cdot \sqrt{3}}{2} = 1 \cdot \sqrt{3}$$

$$\frac{(b+c)^2 - a^2}{bc(\cos A + 1)} \Rightarrow (b+c)^2 - a^2 = b^2 + c^2 + 2bc - a^2$$

$$= (b^2 + c^2 + 2bc) - (b^2 + c^2 - 2bc \cdot \cos A) = 2bc(1 + \cos A)$$

$$\Rightarrow \frac{2bc(1 + \cos A)}{bc(\cos A + 1)} = 2$$

$$\frac{a^2 + b^2 - c^2}{a+b-c} = a \Rightarrow a^2 + b^2 - c^2 = a(a+b-c) \Rightarrow a^2 + b^2 - c^2 = a^2 + ab - ac$$

$$\Rightarrow b^2 - c^2 = ab - ac = a(b-c) \Rightarrow (b-c)(b+c) = a(b-c)$$

$$\Rightarrow (b-c)(b+c+a) = 0 \quad \frac{b \neq c}{b-c \neq 0} \Rightarrow b+c+a = a \Rightarrow b+c = 0$$

$$\Rightarrow b^2 + bc + c^2 = b^2 + c^2 - 2bc \cos A \Rightarrow -2bc \cos A = bc \Rightarrow \cos A = -\frac{1}{2} \Rightarrow A = 120^\circ$$

$$S = 2 \Rightarrow \frac{1}{2} \times (1 + \sqrt{3} \cos \theta) (\sqrt{3} \cos \theta) \times \frac{1}{\sqrt{3}} = 2 \Rightarrow -\sqrt{3} \cos^2 \theta + \cos \theta + \sqrt{3} = 0$$

$$\Rightarrow -\sqrt{3} \cos^2 \theta + \cos \theta + \sqrt{3} = 0 \Rightarrow \cos^2 \theta + \cos \theta + 1 = 0 \Rightarrow (\cos \theta + 1)(\cos \theta + 1) = 0$$

$$\Rightarrow \cos \theta = -1$$

$$\Rightarrow \cos \theta = -1 \Rightarrow a^2 = (1)^2 + (\sqrt{3})^2 - 2(1 \times \sqrt{3} \times \frac{1}{\sqrt{3}})$$

$$\Rightarrow a^2 = 1 + 3 - 2 = 2 \Rightarrow a = \sqrt{2}$$