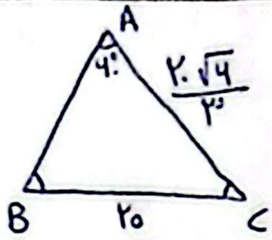
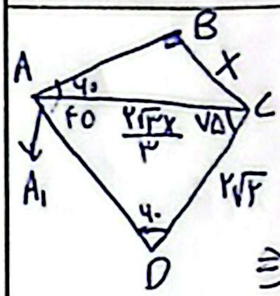
نام و نام خانوادگی ...[illegible]...... پاسخنامه تشریحی تکلیف شماره ...۱۳... کلاس ..............



$\hat A + \underbrace{\hat B + \hat C}_{120^\circ} = 180^\circ \Rightarrow \hat A = 60^\circ$

$\frac{BC}{\sin 60} = \frac{\frac{20\sqrt6}{3}}{\sin B} \Rightarrow \frac{20}{\frac{\sqrt3}{2}} = \frac{\frac{20\sqrt6}{3}}{\sin B}$ ۱

$\Rightarrow \frac{40\sqrt3}{3}\times \sin B = \frac{20\sqrt6}{3} \Rightarrow \sin B = \frac{\sqrt2}{2} \Rightarrow B = 45^\circ , C = 75^\circ$



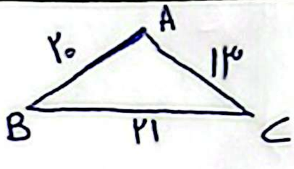
$A_1 = 45$ , $\frac{x}{\sin 60} = \frac{AC}{\sin 90} \Rightarrow AC = \frac{2x}{\sqrt3} = \frac{2\sqrt3 x}{3}$

$\frac{2\sqrt2}{\sin 45} = \frac{\frac{2\sqrt3 x}{3}}{\sin 60} \Rightarrow \frac{2\sqrt2}{\frac{\sqrt2}{2}} = \frac{\frac{2\sqrt3 x}{3}}{\frac{\sqrt3}{2}}$ ۲

$\Rightarrow 4 = \frac{4x}{3} \Rightarrow x = 3$

$(b^2-2)\times \frac{\sin C}{\sin B} = c \Rightarrow \frac{b^2-2}{\sin b} = \frac{c}{\sin C} = \frac{b}{\sin b}$

$\Rightarrow b^2-2=b \Rightarrow b^2-b-2=0 \Rightarrow (b-2)(b+1)=0 \Rightarrow \begin{cases} b=2 \\ b=-1 \end{cases}$ غ ق ق ۳



$AC^2 = AB^2 + BC^2 - 2\,AB\cdot BC\cdot \cos B$

$\Rightarrow 13^2 = 20^2 + 21^2 - 2\times20\times21\times\cos B$

$169 = 400 + 441 - 840\cos B$ ۴

$840\cos B = 672 \Rightarrow \cos B = \frac{672}{840} = 0.8$

$\cos^2 B + \sin^2 B = 1 \Rightarrow 0.64 + \sin^2 B = 1 \Rightarrow \sin^2 B = 0.36 \Rightarrow \sin B = 0.6$

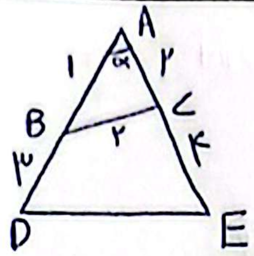
الف) $a^2 = 4^2 + (2+2\sqrt3)^2 - 2\times4\times2(1+\sqrt3)\times\cancel{\cos 60}\ \frac{1}{2}$

$a^2 = 16 + 4 + \cancel{8\sqrt3} + 12 - 8 - \cancel{8\sqrt3}$

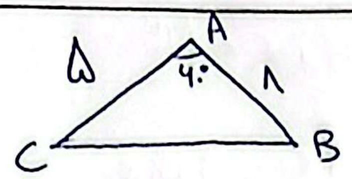
$\Rightarrow a^2 = 24 \Rightarrow a = \sqrt{24} = 2\sqrt6$ ۵

ب) $\frac{4}{\sin B} = \frac{2\sqrt6}{\frac{\sqrt3}{2}} \Rightarrow \frac{4}{\sin B} = 4\sqrt2 \Rightarrow \sin B = \frac{4}{4\sqrt2} =$

$\Rightarrow \sin B = \frac{\sqrt2}{2} \Rightarrow B = 45^\circ , C = 75^\circ$



$$BC^2 = AC^2 + AB^2 - 2AC \cdot AB \cos\alpha$$
$$4 = 4 + 1 - 4\cos\alpha \Rightarrow 4\cos\alpha = 1 \Rightarrow \cos\alpha = \frac{1}{4}$$ ۶
$$DE^2 = AE^2 + AD^2 - 2AE \cdot AD \cos\alpha$$
$$DE^2 = 36 + 16 - 2 \times 4 \times 6 \times \frac{1}{4}$$
$$DE^2 = \sqrt{40} \Rightarrow DE = 2\sqrt{10}$$



الف) $BC^2 = AB^2 + AC^2 - 2AB \cdot AC \times \cos 60^\circ$
$$BC^2 = 64 + 25 - 2 \times 5 \times 8 \times \frac{1}{2}$$
$$\Rightarrow BC^2 = 89 - 40 \Rightarrow BC^2 = 49 \Rightarrow BC = 7$$ ۷

ب) $S = \frac{1}{2} \times AC \times AB \times \sin 60 = \frac{1}{2} \times 8 \times 5 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}$

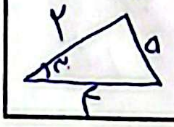
$$a^2 = b^2 + c^2 - 2bc\cos\hat{A}, \quad \frac{(b+c)^2 - a^2}{bc(\cos\hat{A} + 1)} \Rightarrow$$
$(b+c)^2 - a^2 = b^2 + c^2 + 2bc - a^2 \Rightarrow$ حاصل صورت $= \cancel{b^2} + \cancel{c^2} + 2bc - \cancel{b^2} - \cancel{c^2} + 2bc\cos\hat{A}$ ۸

حاصل صورت $= 2bc(1 + \cos\hat{A})$

حاصل عبارت $= \dfrac{2bc\cancel{(1+\cos\hat{A})}}{bc\cancel{(1+\cos\hat{A})}} \Rightarrow$ حاصل عبارت $= 2$

$$\frac{a^3 + b^3 - c^3}{a + b - c} = a^2 \Rightarrow a^3 + b^3 - c^3 = a^3 + a^2 b - a^2 c$$
$$b^3 - c^3 = a^2(b - c)$$
$$a^2 = b^2 + c^2 - 2bc\cos\hat{A} \qquad \cancel{(b-c)}(b^2 + c^2 + bc) = a^2\cancel{(b-c)}$$ ۹
$$\cancel{b^2} + \cancel{c^2} + bc = \cancel{b^2} + \cancel{c^2} - 2bc\cos A \Rightarrow \cos\hat{A} = \frac{-1}{2}$$

$$S = \frac{1}{2} AB \cdot AC \times \underset{\frac{1}{2}}{\sin 30^\circ} \Rightarrow \frac{(3 - \cos\theta)(1 + 3\cos\theta)}{4} = 2$$
$$\Rightarrow -3\cos^2\theta + 8\cos\theta + 3 = 8 \rightarrow 3\cos^2\theta - 8\cos\theta + 5 = 0$$ ۱۰
$$\frac{-b \pm \sqrt{\Delta}}{2a}, \quad \frac{8 \pm \sqrt{64 - 60}}{6} = \cos\theta \rightarrow \cos\theta = \frac{10}{6}, \quad \cos\theta = 1$$



$$a^2 = 2^2 + 4^2 - 2 \times 2 \times 4 \times \frac{\sqrt{3}}{2} \Rightarrow a^2 = 20 - 8\sqrt{3}$$