حسین شیرازی ۲۰

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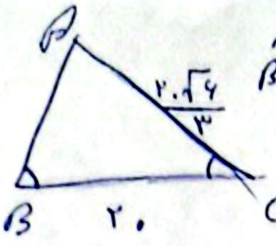

$\hat B + \hat C = 120$ $\quad \hat A = 60$ $\quad AC$

$\frac{\sin\hat B}{\frac{20\sqrt6}{3}} = \frac{\sin 60 = \frac{\sqrt3}{2}}{20} \Rightarrow \sin B = \frac{20\sqrt6}{3} \times \frac{\sqrt3}{2} \times \frac{1}{20}$

$\sin\hat B = \frac{\sqrt2}{2}$ $\quad \frac{3\sqrt2}{3\times 2} =$

$\hat B = 45°$

$C = 120 - 45$ $\quad \hat C = 75°$ $\quad \hat B = 45°$

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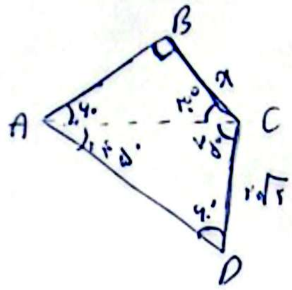

$\hat A_2 = 180 - (60 + 75) = 45°$

$\frac{\sin 45°}{2\sqrt2} = \frac{\sin 60°}{AC} \Rightarrow AC = \frac{2\sqrt2 \times \sin 60°}{\sin 45°} = \frac{2\sqrt2 \times \sqrt3}{2 \times \frac{\sqrt2}{2}} = 2\sqrt3$

$\sin 60° = \frac{x}{AC} \Rightarrow \frac{\sqrt3}{2} = \frac{x}{2\sqrt3}$ $\quad \boxed{x = 3}$

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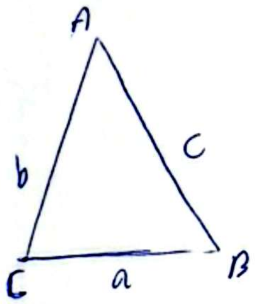

$(b^2 - 2)\frac{\sin\hat C}{\sin\hat B} = \frac{c}{1}$

$\sin\hat C(b^2 - 2) = \sin\hat B\, c$ $\quad \frac{\sin\hat C}{c} = \frac{\sin\hat B}{b^2 - 2}$

$b^2 - 2 = b$ $\quad b^2 - b - 2 = 0$

$(b - 2)(b + 1) = 0$ $\quad b = 2$ قق

$b = -1$ غقق

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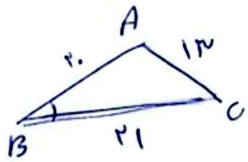

$\sin\hat B = ?$ $\quad 13 = \sqrt{\underbrace{400 + 441}_{841} - 2\cos\hat B\, 420}$

$169 = 841 - 2\cos\hat B\, 420$

$672 = 2\cos\hat B\, 420$ $\quad \cos\hat B = \frac{336}{420} = \frac{8}{10}$

$336 = \cos\hat B\, 420$

$\cos\hat B = \frac{8}{10}$ $\quad \boxed{\sin\hat B = \frac{6}{10}}$

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$b=۴$, $c=۲(۱+\sqrt{۳})$, $\hat{A}=۶۰°$

اندازه ضلع a را بیابید

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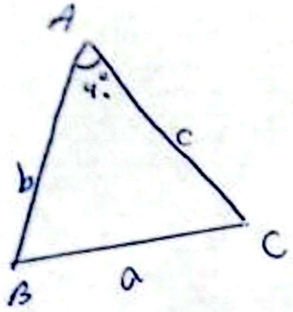
ب)



$a^۲=۱۶+۴(۱+\sqrt{۳})^۲-۸\times\frac{۱}{۲}\times۴\times۲(۱+\sqrt{۳})$

$۱۶+۴(۴+۲\sqrt{۳})$

$۱۶+۱۶+\cancel{۸\sqrt{۳}}-۸-\cancel{۸\sqrt{۳}}=۲۴$ $\quad a^۲=۲۴$ $\quad a=\sqrt{۲۴}$

$\hat{C}=۱۲۰-۴۵$ $\quad \boxed{\hat{C}=۷۵}$

$\sin\hat{B}=\frac{\sqrt{۲}}{۲}$ $\quad \boxed{\hat{B}=۴۵°}$

$\sin\hat{B}=\frac{\sqrt{۳}}{\sqrt{۶}}=\frac{\sqrt{۳}}{\sqrt{۳}\sqrt{۲}}$

$\frac{\sin\hat{A}}{a}=\frac{\sin\hat{B}}{b}=\frac{\sin\hat{C}}{c}\Rightarrow$ $\frac{\sqrt{۳}}{۲}\times۴=۲\sqrt{۶}\times\sin\hat{B}$

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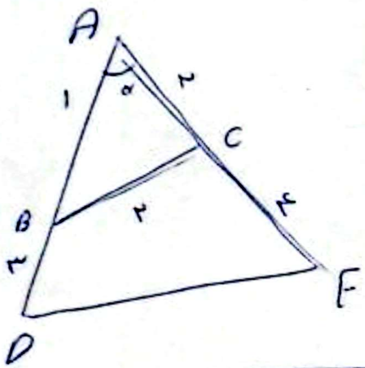

$۲=\sqrt{۱+۴-۴\cos\alpha}$ $\quad ۴=۵-۴\cos\hat{B}$ $\quad \cos\hat{B}=\frac{۱}{۴}$

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$DE=\sqrt{۱۶+۳۶-۲\times\frac{۱}{۴}\times۶\times۴}$ $\quad ۵۲-۱۲$ $\quad \Rightarrow DE=\sqrt{۴۰}$

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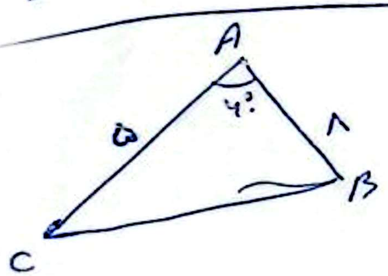

الف)

$\cos۶۰°=\frac{۱}{۲}$ $\quad BC=\sqrt{۲۵+۶۴-۲\times\frac{۱}{۲}\times۴۰}$ $\quad BC=\sqrt{۴۹}$ $\quad BC=۷$

ب)

$\frac{۱}{۲}\times۵\times۸\times\cancel{\sin۶۰}\ \frac{\sqrt{۳}}{۲}=۱۰\sqrt{۳}$

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$$\frac{(b+c)^۲-a^۲}{bc(\cos\hat{A}+۱)} = \frac{b^۲+c^۲+۲bc-b^۲-c^۲+۲\cos\hat{A}\,bc}{bc(\cos\hat{A}+۱)} = \frac{۲bc+۲\cos\hat{A}\,bc}{bc(\cos\hat{A}+۱)} = \frac{۲\cancel{bc}\cancel{(\cos\hat{A}+۱)}}{\cancel{bc}\cancel{(\cos\hat{A}+۱)}} = ۲$$

$$a^۲ = b^۲+c^۲-۲\cos\hat{A}\,bc$$

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$$\frac{a^{۳}+b^{۳}-c^{۳}}{a+b-c}=a^{۲}$$

$$\cancel{a^{۳}}+b^{۳}-c^{۳}=\cancel{a^{۳}}+ba^{۲}-ca^{۲}$$

$$b^{۳}-c^{۳}=a^{۲}(b-c) \qquad \cancel{(b-c)}(b^{۲}+bc+c^{۲})=a^{۲}\cancel{(b-c)}$$

$$a^{۲}=b^{۲}+c^{۲}-۲\cos\hat{A}\,bc$$

$$b^{۲}+c^{۲}+bc=a^{۲}$$

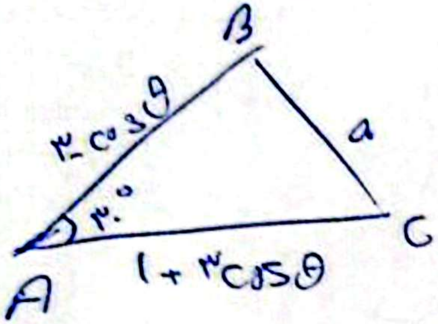
$$b^{۲}+c^{۲}-۲\cos\hat{A}\,bc=a^{۲}$$

$$-۲\cos\hat{A}\,bc=bc$$

$$-۲\cos\hat{A}=۱$$

$$\boxed{\cos\hat{A}=-\frac{۱}{۲}} \qquad \hat{A}=۱۲۰$$

10.



$$a^2 = (3-\cos\theta)^2 + (1+3\cos\theta)^2 - 2\times\frac{\sqrt3}{2}(3-\cos\theta)(1+3\cos\theta)$$

$$10+10\cos^2\theta - 3\sqrt3 - 8\sqrt3\cos\theta + 3\sqrt3\cos^2\theta$$

$$\boxed{a^2 = 10 - 3\sqrt3 + (10+3\sqrt3)\underbrace{\cancel{\cos^2\theta}}_{1} - 8\sqrt3\underbrace{\cancel{\cos\theta}}_{1}} \Rightarrow \boxed{a^2 = 20 - 8\sqrt3}$$

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$$\frac12\times\sin 30^\circ\times(3-\cos\theta)(1+3\cos\theta)=2 \qquad 3-\cos\theta+9\cos\theta-3\cos^2\theta=8$$

$$-3\cos^2\theta+8\cos\theta-5=0 \qquad \cos^2\theta+8\cos\theta+15=0 \qquad \left(\cos\theta+\frac{3}{-3}\right)\left(\cos\theta+\frac{5}{-3}\right)=0$$

$\cos\theta = \frac{5}{3}$ غ ق ق $\qquad \cos\theta = 1$ ق ق