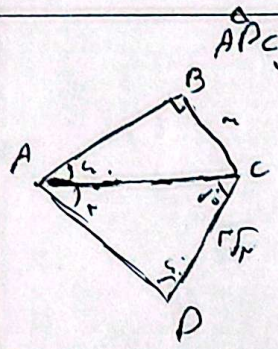


$$\hat{A} + \hat{B} + \hat{C} = 180^\circ \rightarrow \hat{A} = 90^\circ$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{5}{1} = \frac{4}{\sin B} \rightarrow \sin B = \frac{4}{5}$$

$$\hat{B} = 53^\circ$$

$$\hat{A} + \hat{B} + \hat{C} = 180^\circ \rightarrow \hat{C} = 37^\circ$$



$$\hat{A} + \hat{C} + \hat{D} = 180^\circ \rightarrow \hat{A} = 90^\circ$$

$$\frac{AD}{\sin A} = \frac{AC}{\sin D} \rightarrow AD = 3 \sin 37^\circ$$

$$\frac{BD}{\sin B} = \frac{AD}{\sin D} \rightarrow BD = 3 \sin 53^\circ$$

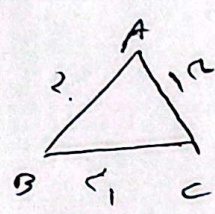
$$BC = BD + DC = 3 \sin 53^\circ + 3 \sin 37^\circ = 3$$

$$(b^2 - r) \frac{\sin C}{\sin B} = c \Rightarrow (b^2 - r) \sin C = c \sin B$$

$$\frac{c}{\sin C} = \frac{b}{\sin B} \Rightarrow c \sin B = b \sin C \quad (I)$$

$$\Rightarrow b^2 - r = b \Rightarrow b^2 - b - r = 0$$

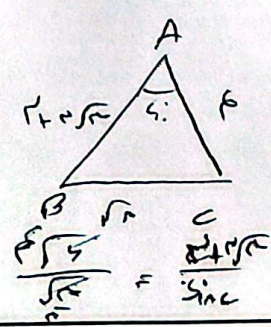
$$b = \frac{1 \pm \sqrt{1+4r}}{2}$$



$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$16 = 25 + 9 - 30 \cos B \Rightarrow \cos B = \frac{18}{30} = \frac{3}{5}$$

$$\sin B = \frac{4}{5}$$



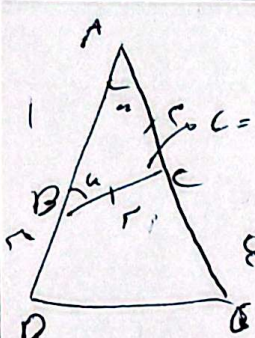
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$25 = 16 + 9 - 24 \cos A \Rightarrow \cos A = \frac{1}{4}$$

$$\sin A = \frac{\sqrt{15}}{4}$$

$$\frac{\sqrt{15}}{\sin A} = \frac{4}{\sin B} \rightarrow \sin B = \frac{4 \sin A}{\sqrt{15}} = \frac{4 \cdot \frac{\sqrt{15}}{4}}{\sqrt{15}} = 1 \rightarrow \hat{B} = 90^\circ$$

$$\hat{C} = 90^\circ$$



$ABC \rightarrow \frac{a}{\sin A} = \frac{b}{\sin B} + \frac{r}{\sin u} = \frac{1}{\sin(180^\circ - r_u)}$ (2)
 $\Rightarrow \frac{r}{\sin u} = \frac{1}{\sin r_u} + \frac{r}{\sin u} = \frac{1}{r \sin r_u} + \cos u = \frac{1}{r}$ ✓
 $DE^2 = b^2 + c^2 - 2bc \cos A \rightarrow DE^2 = r^2 + r^2 - 2r^2 \cos A$
 $DE^2 = \epsilon \rightarrow DE = r\sqrt{1 - \cos A}$ ✓

$a^2 = b^2 + c^2 - 2bc \cos A$ (الف)
 $\epsilon^2 = r^2 + r^2 - 2r^2 \cos A$
 $\epsilon^2 = 2r^2(1 - \cos A)$
 $\epsilon = \sqrt{2}r \sqrt{1 - \cos A}$
 $S = \frac{1}{2} \times \epsilon \times \frac{\epsilon}{\sqrt{2}} = \frac{1}{2} r^2 \sqrt{1 - \cos A}$ (ب)

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\frac{b^2 + c^2 + 2bc - b^2 - c^2 + 2bc \cos A}{bc (\cos A + 1)} = \frac{2bc \cos A + 2bc}{bc (\cos A + 1)} = \frac{2bc (\cos A + 1)}{bc (\cos A + 1)} = 2$$
 (ج)

$$a^2 + b^2 - c^2 = a^2(a + b - c) = a^2 + a^2(b - c)$$

$$b^2 - c^2 = a^2(b - c) \rightarrow (b - c)(b^2 + c^2 + bc) = a^2(b - c)$$

$$\begin{cases} a^2 = b^2 + c^2 + bc \\ a^2 = b^2 + c^2 - 2bc \cos A \end{cases} \rightarrow \cos A = \frac{1}{2} \rightarrow A = 120^\circ$$
 (د)

$$S_{ABC} = \frac{1}{2} \times \frac{1}{2} (1 + \sqrt{3} \cos \theta)(1 - \cos \theta) = \frac{1}{2}$$

$$1 - \sqrt{3} \cos \theta - \cos^2 \theta + \sqrt{3} \cos^2 \theta - \cos \theta + \frac{1}{2} = 0$$

$$\cos^2 \theta - \cos \theta + \frac{1}{2} = 0 \rightarrow \begin{cases} \cos \theta = 1 \checkmark \\ \cos \theta = \frac{1}{2} \times \end{cases}$$
 (هـ)

$$a^2 = r^2 + r^2 - 2 \times r \times \frac{\sqrt{3}}{2} \times r \rightarrow a^2 = r^2 - 1\sqrt{3}r$$