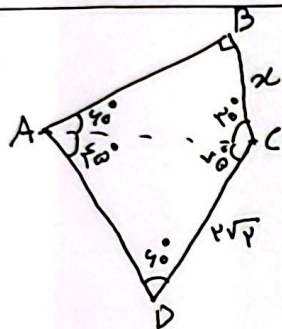


$$B + C + A = 180 \rightarrow A = 40^\circ$$

$$\frac{20}{\sin A} = \frac{20\sqrt{2}}{\sin B} \rightarrow \sin B = \frac{20\sqrt{2} \times \sqrt{2}}{20 \times 2} = \frac{2 \times 2}{2 \times 2} = \frac{\sqrt{2}}{2} \rightarrow \hat{B} = 45^\circ$$

$$C + B = 120 \rightarrow C + 45 = 120 \rightarrow C = 75^\circ$$



$$\hat{A} + \hat{B} + \hat{C} + \hat{D} = 360 \rightarrow \hat{A} + 90 + 40 + 100 = 360 \rightarrow \hat{A} = 100^\circ$$

$$\frac{2\sqrt{2}}{\sin 100} = \frac{AC}{\sin 40} \rightarrow \frac{2\sqrt{2}}{\frac{\sqrt{2}}{2}} = \frac{AC}{\frac{\sqrt{2}}{2}} \rightarrow AC = 2\sqrt{2}$$

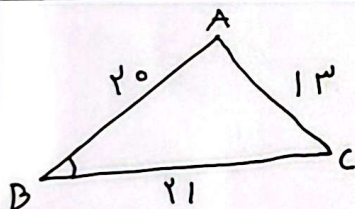
$$\sin 40 = \frac{x}{AC} = \frac{\sqrt{2}}{2} \rightarrow \frac{x}{2\sqrt{2}} = \frac{\sqrt{2}}{2} \rightarrow x = 2$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k \rightarrow b = k \sin B, c = k \sin C$$

$$\frac{(b^2 - 2) \sin C}{\sin B} = c \rightarrow \frac{cb^2 - 2}{\sin B} = k \sin C \rightarrow b^2 - 2 = k \sin B = b$$

$$\rightarrow b^2 - b - 2 = 0 \rightarrow b = 2 \text{ or } b = -1$$

دو ضلع نمی تواند منفی باشد

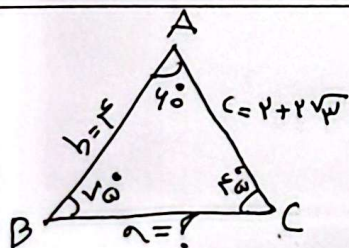


$$AC^2 = BC^2 + BA^2 - 2BA \cdot BC \cos B$$

$$12^2 = 11^2 + 20^2 - 2 \times 11 \times 20 \cos B$$

$$144 = 121 + 400 - 440 \cos B \rightarrow 440 \cos B = 457 \rightarrow \cos B = 0.11$$

$$\cos^2 B + \sin^2 B = 1 \rightarrow \frac{14}{40} + \sin^2 B = 1 \rightarrow \sin^2 B = \frac{9}{40} \rightarrow \sin B = \frac{3}{\sqrt{40}}$$

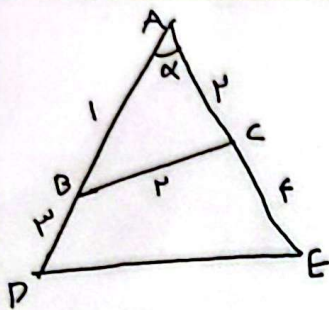


$$a = \sqrt{AB^2 + AC^2 - 2AB \cdot AC \cos 40} = \sqrt{f^2 + (2 + 2\sqrt{2})^2 - 2f(2 + 2\sqrt{2}) \cos 40}$$

$$= \sqrt{f^2} = f$$

$$\frac{2\sqrt{2}}{\sin 40} = \frac{f}{\sin C} \rightarrow \frac{2\sqrt{2}}{\frac{\sqrt{2}}{2}} = \frac{f}{\sin C} \rightarrow \sin C = \frac{\sqrt{2}}{2} \rightarrow \hat{C} = 45^\circ$$

$$\hat{C} + \hat{A} + B = 180 \rightarrow 45 + 40 + B = 180 \rightarrow B = 95^\circ$$

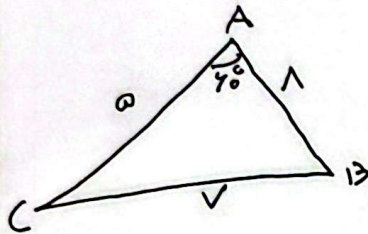


$$BC^2 = AB^2 + AC^2 - 2ABAC \cos \alpha$$

$$f = f + 1 - f \cos \alpha \rightarrow \cos \alpha = \frac{1}{f} \quad \checkmark$$

$$DE^2 = AD^2 + AE^2 - 2ADAE \cos \alpha = 1 + 1 - f \times \frac{1}{f}$$

$$= f_0 \rightarrow DE = \sqrt{f_0} = \sqrt{10} \quad \checkmark \quad (y)$$

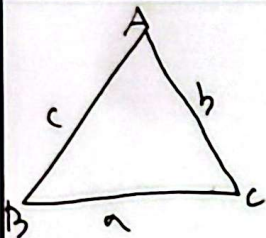


$$BC^2 = AC^2 + AB^2 - 2ACAB \cos 40^\circ$$

$$= 1 + 1 - 2 \times 1 \times 1 \times \cos 40^\circ = f_0 \rightarrow BC = \sqrt{f_0} \quad \checkmark \quad (y)$$

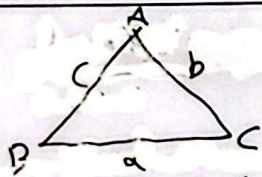
$$S = \frac{1}{2} \times AC \times AB \times \sin 40^\circ$$

$$= \frac{1}{2} \times 1 \times 1 \times \sin 40^\circ = \frac{1}{2} \sin 40^\circ \quad \checkmark \quad (y)$$



$$\frac{(b+c)^2 - a^2}{bc \cos A + 1} = \frac{b^2 + c^2 + 2bc - a^2}{bc \cos A + 1} = \frac{b^2 + c^2 + 2bc - (b^2 + c^2 - 2bc \cos A)}{bc \cos A + 1}$$

$$= \frac{2bc + 2bc \cos A}{bc \cos A + 1} = \frac{2bc(1 + \cos A)}{bc \cos A + 1} = 2 \quad \checkmark \quad (y)$$



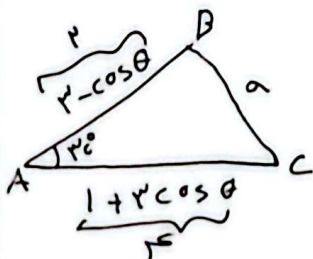
$$\frac{a^2 + b^2 - c^2}{a + b - c} = a \rightarrow a^2 + b^2 - c^2 = a^2 + a^2 b - a^2 c$$

$$\rightarrow b^2 - c^2 = a^2(b - c) \rightarrow b^2 + c^2 + bc = a^2$$

$$(b+c)(b^2 + c^2 + bc) = a^2$$

$$b^2 + c^2 + bc = b^2 + c^2 - 2bc \cos A$$

$$\rightarrow \cos A = -\frac{1}{2} \rightarrow A = 120^\circ \quad \checkmark \quad (y)$$



$$S = \frac{1}{2} \times AB \times AC \times \sin \theta \rightarrow \frac{1}{2} \times 1 \times 1 \times \sin \theta = \frac{1}{2} \times (1 - \cos \theta) \times (1 + \cos \theta)$$

$$\rightarrow \sin \theta = 1 - \cos^2 \theta \rightarrow 1 - \cos^2 \theta = 1 - \cos^2 \theta$$

$$\rightarrow 1 - \cos^2 \theta = 1 - \cos^2 \theta \rightarrow 1 - \cos^2 \theta = 1 - \cos^2 \theta$$

$$\omega_{\theta,1} = \frac{1 \pm 1}{2} \rightarrow \frac{1}{2} = \cos \theta \times \frac{1}{2} \rightarrow 1 = \cos \theta \quad \checkmark \quad (y)$$

$$a^2 = AB^2 + AC^2 - 2ABAC \cos \theta = 1 + 1 - 2 \times 1 \times 1 \times \cos \theta = 2 - 2 \cos \theta \quad \checkmark \quad (y)$$