

$$B(1, \tan \theta) \quad AB^2 = (1 - \cos \theta)^2 + (\tan \theta - \sin \theta)^2 \rightarrow AB = \frac{1 - \cos \theta}{\cos \theta}$$

$$A(\cos \theta, \sin \theta)$$

$$\alpha = \frac{\pi}{2} - \theta \rightarrow \sin \alpha = \cos \theta \rightarrow AB = \frac{1 - \sin \alpha}{\sin \alpha}$$

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$$\frac{BD}{DC} = r = \frac{y}{x} = \frac{AB \sin \alpha}{AC \sin \alpha} = \frac{\frac{r}{x} AB}{\frac{1}{x} AC} = r \Rightarrow \frac{AB}{AC} = r \rightarrow \boxed{\frac{AB}{AC} = r}$$

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$$x_A = \frac{1}{r}, y_A = \sqrt{1 - (\frac{1}{r})^2} = \frac{\sqrt{r^2 - 1}}{r}$$

$$x_B = \frac{1}{r}, y_B = -\frac{\sqrt{r^2 - 1}}{r}$$

$$x_C = 0, y_C = -1$$

$$AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} = \sqrt{(\frac{1}{r} - \frac{1}{r})^2 + (-\frac{\sqrt{r^2 - 1}}{r} - \frac{\sqrt{r^2 - 1}}{r})^2} = \sqrt{r^2 - 1}$$

$$BC = \sqrt{(\frac{1}{r} - 0)^2 + (-\frac{\sqrt{r^2 - 1}}{r} + 1)^2} = \sqrt{r^2 - 1}$$

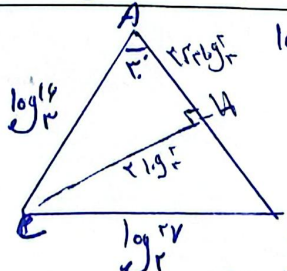
$$CA = \sqrt{(\frac{1}{r} - 0)^2 + (\frac{\sqrt{r^2 - 1}}{r} + 1)^2} = \sqrt{r^2 + 1}$$

زاویه A ۴۰ درجه است و B ۵۰ درجه است اختلاف آن‌ها ۹۰ درجه است بنابراین زاویه C که مکملی است ۹۰ درجه است

$$S = \frac{1}{2} \sin \alpha \times BC \times AC = \frac{\sqrt{r^2 - 1}}{r} \times \sqrt{r^2 + 1}$$

$$P = AB + BC + CA = \sqrt{r^2 - 1} + \sqrt{r^2 - 1} + \sqrt{r^2 + 1}$$

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$$\log \frac{14}{7} = 2 \log 7$$

$$\log \frac{16}{8} = 2 \log 8$$

$$\log \frac{17}{1} = 3 \log 7$$

$$AH = 2 \log 7 \times \cos \alpha = 2r \log 7$$

$$CH = \sqrt{(2 \log 7)^2 - (2r \log 5)^2} = 2 \log 5$$

$$BH = \sqrt{(3 \log 7)^2 - (2r \log 5)^2} = \frac{\sqrt{9 - 4(\log 5)^2}}{\log 5}$$

$$S = \frac{1}{2} \sin \alpha \times AB \times AC$$

$$S = \frac{1}{2} \times \sin \alpha \times \log \frac{14}{7} \times \log \frac{16}{8} = \frac{1}{2} \times \sin \alpha \times \log \frac{17}{1} \times \log \frac{14}{7}$$

$$S_{ABC} = ?$$

$$S = \sqrt{9 - 4(\log 5)^2} + 2r \log 5$$

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$$x = \cos \alpha \Rightarrow f(\cos \alpha) = 2 \cos^2 \alpha - \cos \alpha - \sin^2 \alpha (r \sin^2 \alpha - 1) = 2 \cos^2 \alpha - \cos \alpha - r \sin^2 \alpha + \sin^2 \alpha$$

$$= \frac{2(\cos^2 \alpha - \sin^2 \alpha)}{(\sin^2 \alpha + \cos^2 \alpha) / (\cos^2 \alpha - \sin^2 \alpha)} - (\cos^2 \alpha - \sin^2 \alpha) = 2(\cos^2 \alpha - \sin^2 \alpha) - (\cos^2 \alpha - \sin^2 \alpha)$$

$$= \cos^2 \alpha - \sin^2 \alpha = \cos 2\alpha = 1 \rightarrow \sin 2\alpha = 0$$

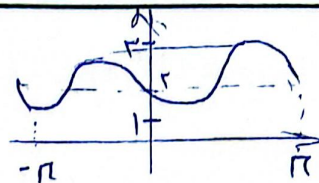
$$\cos 2\alpha + \sin^2 \alpha = 1$$

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$$a) \Rightarrow y = -\sin 2x + 2 \quad T = \frac{2\pi}{|b|} = \pi$$

$$[-\pi, \pi]$$

با نقطه‌های منفرجه را رسم می‌کنیم

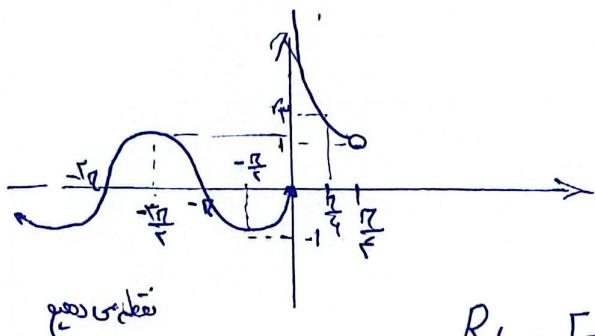
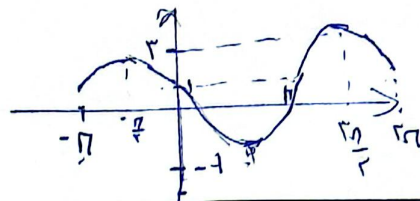


$$b) y = 2\cos(x + \frac{\pi}{4}) + 1$$

$$= 2\cos(x + \frac{\pi}{4}) + 1$$

$$[-\pi, \pi]$$

$$T = \frac{2\pi}{|a|} = 2\pi$$



$$R_f = [-1, +\infty)$$

$$f(x) = a + b \sin(cx) \cos(cx) \cos(2cx) = a + b \frac{1}{4} \sin(4cx) = 2 + 2 \sin 4x$$

$$\frac{1}{4} \sin(4cx)$$

$$\frac{1}{4} \sin(4cx)$$

$$T = \frac{2\pi}{|b|} = \frac{2\pi}{0} \Rightarrow |c| = 0 \Rightarrow c = \frac{0}{4}$$

$$T = \frac{\pi}{4} - \frac{\pi}{4} = \frac{\pi}{2}$$

$$\Rightarrow \frac{b}{a} = \frac{1 \times \frac{\pi}{4}}{2} = \frac{0}{2}$$

$$max = |a| + b = 4$$

$$min = -|a| + b = 0$$

$$a = 2$$

$$b = \pm 1$$

$$b = 1$$

$$max = |a| + c = 2$$

$$min = -|a| + c = -2$$

$$\Rightarrow c = -1$$

$$a = \pm 2 \rightarrow a = -2$$

$$T = \pi$$

$$T = \frac{2\pi}{|b|} = \pi \Rightarrow |b| = 2 \Rightarrow b = \pm 2$$

$$f(x) = -2\cos 2x - 1 \rightarrow f(a) = -2\cos 2a - 1 = 0 \rightarrow \cos 2a = -\frac{1}{2}$$

$$\sin 2a = 1 - \cos 2a = 1 - (-\frac{1}{2}) = \frac{3}{2} \Rightarrow \sin 2a = \frac{3}{2} \rightarrow |tan 2a| = \left| \frac{\frac{3}{2}}{-\frac{1}{2}} \right| = 3$$

$$T = 3$$

$$\rightarrow f(x+3) = f(x)$$

$$x = -3t, 0 + 2k \rightarrow -1 \sqrt{-3t, 0 + 2k} \sqrt{2} \rightarrow k = 1 \rightarrow x = -3t, 0 + 2k = 1, 0$$

$$-1, 1, 2, 3$$

$$\rightarrow f(-3t, 0) = f(1, 0) \rightarrow f(1, 0) = -\frac{1}{3}x + 1 = -\frac{1}{3}(\frac{1}{2}) + 1 = -\frac{1}{6} + 1 = \frac{5}{6}$$