محمد ایما دعوتی — دوازدهم ریاضی — 14,75

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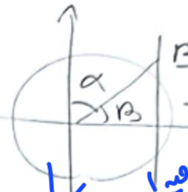

$BT,\ \tan B = \cot\alpha$

$OT = 1$

$OB = \frac{1}{\sin\alpha} \leftarrow OB>0 \quad OB = \left|\frac{1}{\sin\alpha}\right|$

$AB = OB - 1 \Rightarrow \left|\frac{1}{\sin\alpha}\right| - 1 \rightarrow \frac{1}{\sin\alpha} - 1$

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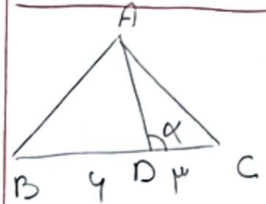

$\frac{AD}{\sin B} = \frac{BD}{\sin A} = \frac{6}{\frac{\sqrt2}{2}} \Rightarrow \frac{\sin B}{\sin C} = \frac{\frac{AD}{6\sqrt2}}{\frac{AD}{6}} = \frac{\sqrt2}{2}$

$\frac{AD}{\sin C} = \frac{DC}{\sin A} = 6$

$\frac{AB}{\sin C} = \frac{AC}{\sin B} \Rightarrow \frac{\sin C}{\sin B} = \frac{AB}{AC} = \boxed{\sqrt2}$

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$\sqrt{\left(\frac12 - \left(-\frac{\sqrt3}{2}\right)\right)^2 + \left(\frac{\sqrt3}{2} - \frac12\right)^2} = \sqrt2$

$\sqrt{\left(\frac12 - 0\right)^2 + \left(\frac{\sqrt3}{2} + 1\right)^2} = \sqrt{2+\sqrt3}$

$\sqrt{\left(-\frac{\sqrt3}{2} - 0\right)^2 + \left(\frac12 - (-1)\right)^2} = \sqrt3$

محیط $= \sqrt2 + \sqrt3 + \sqrt{2+\sqrt3}$

$S = \frac12 \begin{vmatrix} 1 & 1 & 1 \\ \frac12 & -\frac{\sqrt3}{2} & 0 \\ \frac{\sqrt3}{2} & \frac12 & -1 \end{vmatrix} = \boxed{\frac{3+\sqrt3}{4}}$

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با ترسیم ○ می‌بریم

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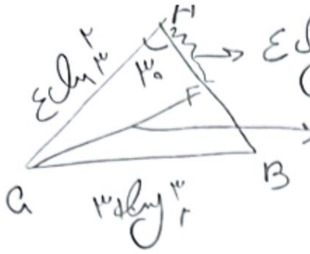

$\rightarrow 4\cos^2\frac{\pi}{3} \times \cos 30 = 2\cos^2\frac{\pi}{3}\sqrt3$

$\rightarrow \sqrt{\left(4\cos^2\frac{\pi}{3}\right)^2 - \left(2\sqrt3\cos^2\frac{\pi}{3}\right)^2} = 2\cos^2\frac{\pi}{3}$

$HB = \frac{CH^2}{AH} \rightarrow \frac{\left(4\cos^2\frac{\pi}{3}\right)^2}{2\sqrt3\cos^2\frac{\pi}{3}} = \frac{8\sqrt3}{3}\cos^2\frac{\pi}{3}$

$S = \frac12 \times 4\cos^2\frac{\pi}{3} \times \frac{8\sqrt3}{3}\cos^2\frac{\pi}{3} \times \frac12$

$\Rightarrow \frac{8\sqrt3}{3}\left(\cos^2\frac{\pi}{3}\right)^2$

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$x \rightarrow \cos$

$2\cos^4 - \cos^2 - \sin^2(2\sin^2 - 1)$

$\Rightarrow \cos^2(\underbrace{2\cos^2 - 1}_{\cos2\alpha}) - \sin^2(\underbrace{2\sin^2 - 1}_{-\cos2\alpha})$

$\Rightarrow \cos2\alpha(\cos^2 + \sin^2) = 1 \Rightarrow \cos2\alpha = 1$

$\Rightarrow 2\alpha = 0 \rightsquigarrow \boxed{\sin2\alpha = 0}$

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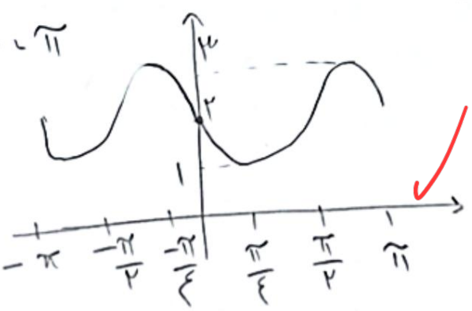
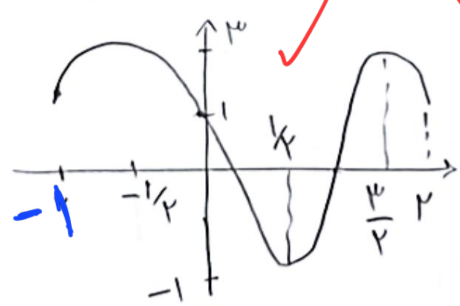
الف) $T = \frac{2\pi}{2} = \pi$



ب) $T = \frac{2\pi}{\pi} = 2$



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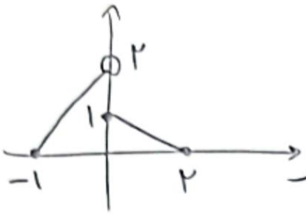

چون در بازه $(-1,2]$ دارای تناوب ۳ می‌باشد!

$-34/5 + 33 = -1/5 + 3 = 1/5 \to f(1/5) = ?$

$f(1/5) = (-\frac{1}{2} \times 1/5) + 1 = \boxed{\frac{1}{4}}$

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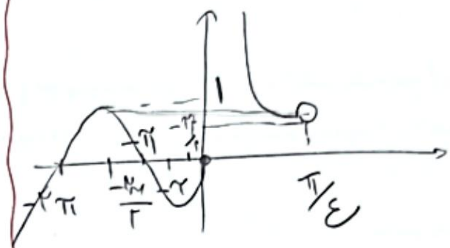
$f(x) = \begin{cases} \cot x & 0 < x < \frac{\pi}{4} \\ \sin & x \leq 0 \end{cases} \to R_f = (-1, +\infty)$



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$f(x) = a + b\underbrace{\sin(cx)\cos(cx)}_{\frac{1}{2}\sin(2cx)}\cos(2cx)$

$\Rightarrow \frac{1}{4}\sin 4cx \qquad f(x) = 2 + 2\sin x$

$T = \frac{2\pi}{4c} = \frac{2\pi}{5} \to c = \frac{5}{4}$

$a + \frac{b}{4}\sin x \to a + \frac{b}{4} = 4 \to 4a + b = 16$

$\to a - \frac{b}{4} = 0 \to a = \frac{b}{4}$

$\Rightarrow a = 2, b = 8$

$\frac{bc}{a} = \frac{\frac{5}{4} \times 8}{2} = \boxed{5}$

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$a\cos bx + c$

$T = \frac{2\pi}{|b|} = \pi \to b = \pm 2$

$\cos = 1 \to a + c = -4$

$\cos = -1 \to c - a = 2$

$\Rightarrow a = -3, c = -1$

$a\cos 2\alpha + c = 0 \to -3\cos 2\alpha - 1 = 0$

$\cos 2\alpha = -\frac{1}{3} \to 1 + \tan^2 2\alpha = \frac{1}{(-\frac{1}{3})^2}$

$1 + \tan^2 2\alpha = 9 \to \tan^2 2\alpha = 8$

$|\tan 2\alpha| = \boxed{2\sqrt{2}}$

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