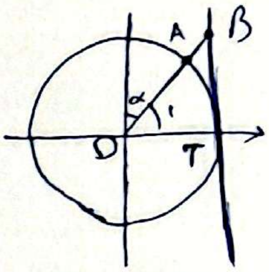
آراد آزادیا — تکلیف شماره ۱۴ — گروه دوازدهم ریاضی A

14,5

(۱)



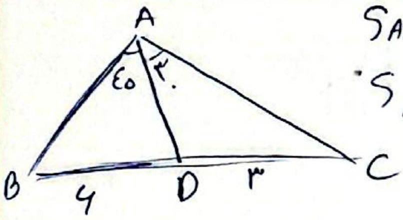
$O_1 + \hat\alpha = \frac{\pi}{2} \Rightarrow \sin\hat\alpha = \cos\hat O_1$

$\cos\hat O_1 = \frac{OT}{OB} = \frac{OT}{AO+AB} \longrightarrow \cos\hat O_1 = \frac{1}{AB+1}$

$\sin\alpha = \frac{1}{1+AB} \Rightarrow 1+AB = \frac{1}{\sin\alpha} \Rightarrow AB = \frac{1-\sin\alpha}{\sin\alpha}$

2

(۲)



$S_{ADC} = \frac13 S_{ABC}\,,\ S_{ADC} = \frac12 AD\cdot AC\cdot\sin 30^\circ$

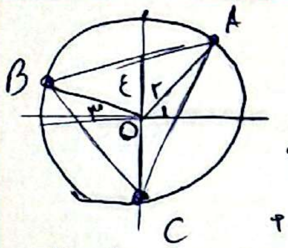
$S_{ABD} = \frac23 S_{ABC}\,,\ S_{ABD} = \frac12\cdot AD\cdot AB\cdot\sin 45$

$\Big\}\ S_{ADC} = \frac12 S_{ABD}$

$\Rightarrow \dfrac{\cancel{\frac12} AD\cdot AB\cdot \cancel{\sin 45}^{\frac{\sqrt2}{2}}}{\cancel{\frac12}\cdot AD\cdot AC\cdot \cancel{\sin 30}^{\frac12}} \Rightarrow \frac{AB}{AC} = \sqrt2$

2

(۳) الف) S_{ABC} ?



1,5

A طول نقطه $= \frac12 \Rightarrow O_1 = 60^\circ\,,\ O_2 = 30^\circ$

B عرض نقطه $= \frac12 \Rightarrow O_3 = 30^\circ\,,\ O_4 = 60^\circ$

$S_{ABC} = S_{OBC} + S_{OBA} + S_{OCA} = \frac12\times BO\times OC\times\sin 120^\circ + \frac12\times BO\times OA\times\sin 90^\circ + \frac12\sin 150^\circ\times OA\times OC \Rightarrow \frac12\times1\times1\times\frac{\sqrt3}{2} + \frac12\times1\times1\times1 + \frac12\times1\times1\times\frac12 = \frac{\sqrt3}{4} + \frac12 + \frac14$

ب) $2P_{ABC} = ?$

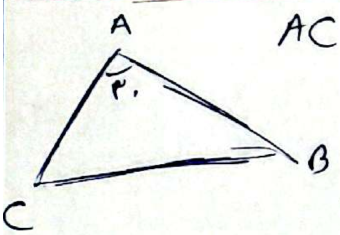
$BC^2 = OB^2 + OC^2 - 2OB\times OC(\cos 120^\circ) \Rightarrow BC = 1+1-2(1)(1)(\frac12) = 1\,,\ AB = \sqrt2$

$AC = OA^2 + OC^2 - 2OA\times OC(\cos 150^\circ) \Rightarrow AC = 1+1-2(1)(1)(\frac{\sqrt3}{2}) = 2\sqrt3$

$2P = 1 + 2 - \sqrt{2+\sqrt2}$

$P = \sqrt2 + \sqrt3 + \sqrt{2+\sqrt3}$

(۴)



$AC = \log_3^{14}\quad BC = \log_2^{27}\quad c^2 = a^2 + b^2 - 2ab\cos 30^\circ \Rightarrow b^2 - a\sqrt3 b + (a^2 - c^2) = 0$

$\Rightarrow b = \dfrac{a\sqrt3 + \sqrt{4c^2 - a^2}}{2} \Rightarrow S = \frac12 ba\sin 30^\circ = \frac{ab}{4}$

$\Rightarrow S = \dfrac{a(a\sqrt3 + \sqrt{4c^2 - a^2})}{2} \Rightarrow a = 4\log_3^2$

2

$\Rightarrow 4c^2 - a^2 = \frac{36}{a^2} - 16a^2 = \dfrac{4(9 - 4a^4)}{a^2} \times \sqrt{4c^2 - a^2} = \dfrac{(2\sqrt{9 - 4a^4}}{a} \Rightarrow S = 2\sqrt3(\log_3^2)^2 + \sqrt{9 - 4\log_3^2}$

(۵)

$2\sin 4x - \sin 2(\alpha)(2\sin^2\alpha - 1) \xrightarrow{\cos\alpha \text{ بطبق}} 2\cos^4 - \cancel{4}\cos^2 - 2\sin^4\alpha + \sin^2\alpha = 1$

$\Rightarrow 2(\underbrace{\cos^4\alpha - \sin^4\alpha}) + \sin^2\alpha - \cos^2\alpha = 1$

$(\underbrace{\cancel{\cos^2\alpha + \sin^2\alpha}}_{1})(\cos^2\alpha - \sin^2\alpha)$

$\Big\}\ 2\cos^2\alpha - 2\sin^2\alpha + \sin^2\alpha - \cos^2\alpha = 1$

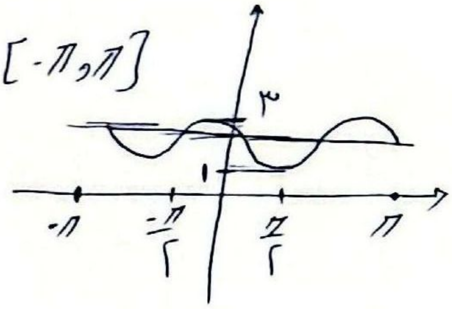
2

$\Rightarrow \cos^2\alpha - \sin^2\alpha = 1 \Rightarrow \cos 2\alpha = 1 \Rightarrow \sin 2\alpha = 1 - \underbrace{\cos^2 2\alpha}_{1} = 0$

(۶)

الف) $-2\sin x\cos x+2$ $\quad[-\pi,\pi]$

$\underbrace{\sin x\cos x}_{\frac{1}{2}\sin 2x}$

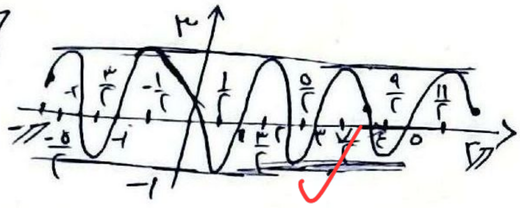


$T=\frac{2\pi}{2}=\pi$

ب) $2\cos(\underbrace{x+\frac{1}{2}}_{})\pi+1$ $\quad[-\pi,2\pi]$

Sin زاویه متمم => تغییر نماد $\frac{\pi}{x}+\frac{\pi}{2}$ $\qquad T=\frac{2\pi}{\pi}=2$

$-2\sin\pi x+1$



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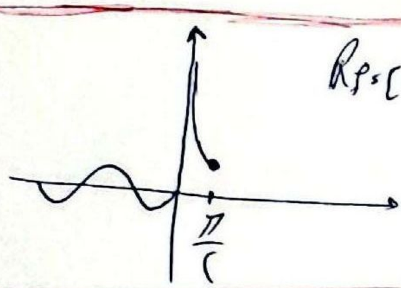

$R_f=[-1,+\infty)$ $\qquad f(x)=\begin{cases}\cot x\ ; & 0<x<\frac{\pi}{4}\\ \sin x\ ; & x\le 0\end{cases}$

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(۸)

$a+b\sin(cx)\cos(cx)\cos(2cx)=a+\frac{b}{2}\sin(2cx)\cos(2cx)=a+\frac{b}{4}\sin 4cx$

$\Rightarrow f(x)=a+\frac{b}{4}\sin(4cx)\Rightarrow T=\frac{5\pi}{10}-\frac{\pi}{10}=\frac{4\pi}{10}=\frac{2\pi}{\omega}=\frac{2\pi}{4c}\Rightarrow\omega=4c\Rightarrow c=\frac{5}{4}$

$\left.\begin{array}{l}a+\frac{b}{4}=4\\ a-\frac{b}{4}=0\end{array}\right\}\xrightarrow{+}2a=4\Rightarrow a=2,\ 2+\frac{b}{4}=4\Rightarrow\frac{b}{4}=2\Rightarrow b=8\Rightarrow\frac{bc}{a}=\frac{\frac{5}{4}\times 8}{2}=5$

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(۹)

$f(x)=a\cos bx+c$ نمودار cos از ابتدا شیب مثبت دارد => a و b مختلف علامت ، $a<0$ و $b>0$ => $T=\frac{2\pi}{b}=\pi$

$\Rightarrow \underline{b=2}$ ، $\left.\begin{array}{l}max=-a+c=2\\ min=a+c=-4\end{array}\right\}\xrightarrow{+}2c=-2\Rightarrow\underline{c=-1}$ ، $a-1=-4\Rightarrow\underline{a=-3}$

$f(x)=-3\cos 2x-1\underset{\text{ریشه‌یابی}}{\Rightarrow}-3\cos2x-1=0\Rightarrow\cos 2x=-\frac{1}{3}$, $1+\tan^2 2x=\frac{1}{\cos^2 2x}\Rightarrow 1+\tan^2 2x=9$

$\Rightarrow\tan^2 2x=8\Rightarrow\tan 2x=\sqrt{8}$

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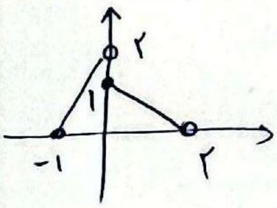

$T=3\Rightarrow f(x)=f(x+3k)\Rightarrow f(-34.5)=f(1.5-36)$

$\Rightarrow f(\underbrace{1.5-3(12)}_{-34.5})=f(1.5)$ $\qquad f(x)=-\frac{1}{2}x+1\Rightarrow f(1.5)=-0.75+1=0.25$

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