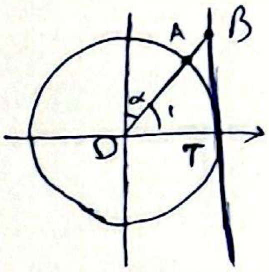
گروه دوازدهم پیران A — تکلیف شماره ۱۴ — آرکا آزادیا

۱)



$O_1 + \hat\alpha = \frac{\pi}{2} \Rightarrow \sin\hat\alpha = \cos\hat O_1$

$\cos\hat O_1 = \frac{OT}{OB} = \frac{OT}{AO+AB} \longrightarrow \cos\hat O_1 = \frac{1}{AB+1}$

$\sin\alpha = \frac{1}{1+AB} \Rightarrow 1+AB = \frac{1}{\sin\alpha} \to AB = \frac{1-\sin\alpha}{\sin\alpha}$

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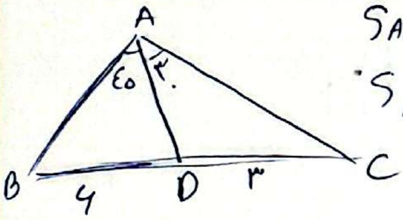
۲)



$S_{ADC} = \frac13 S_{ABC}, \; S_{ADC} = \frac12 AD\cdot AC\cdot\sin 30°$ $\Big\}\; S_{ADC} = \frac12 S_{ABD}$

$S_{ABD} = \frac23 S_{ABC}, \; S_{ABD} = \frac12 \cdot AD\cdot AB\cdot\sin 45$

$\Rightarrow \frac{\cancel{\frac12} AD\cdot AB\cdot \cancel{\sin 45}^{\frac{\sqrt2}{2}}}{\cancel{\frac12}\cdot AD\cdot AC\cdot \cancel{\sin 30}_{\frac12}} = 2 \Rightarrow \frac{AB}{AC} = \sqrt2$

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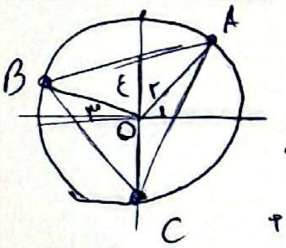
۳) الف) $S_{ABC} = ?$



A طول نقطه $= \frac12 \Rightarrow O_1 = 60°, \; O_2 = 30°$

B عرض نقطه $= \frac12 \Rightarrow O_3 = 30°, \; O_4 = 60°$

$S_{ABC} = S_{OBC} + S_{OBA} + S_{OCA} = \frac12 \times BO\times OC\times\sin 120° + \frac12\times BO\times OA\times\sin 90° + \frac12\sin 150°\times A \times OC \Rightarrow \frac12\times1\times1\times\frac{\sqrt3}{2} + \frac12\times1\times1\times1 + \frac12\times1\times1\times\frac12 = \underline{\frac{\sqrt3}{4} + \frac12 + \frac14}$

ب) $2P_{ABC} = ?$

$BC^2 = OB^2+OC^2-2OB\cdot OC(\cos 120°) \Rightarrow BC = \sqrt{1+1-2(1)(1)(\frac12)} = 1 \quad , AB = \sqrt2$

$AC^2 = OA^2+OC^2-2OA\cdot OC(\cos 150°) \Rightarrow AC = \sqrt{1+1-2(1)(1)(\frac{\sqrt3}{2})} = 2\sqrt3$

$2P = 1+2-\sqrt{2+\sqrt2}$

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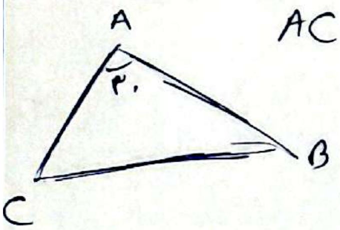
۴)



$AC = \log_3^{16} \quad BC = \log_2^{27} \quad c^2 = a^2+b^2-2ab\cos 30° \Rightarrow b^2 - a\sqrt3\, b + (a^2-c^2) = 0$

$\Rightarrow b = \frac{a\sqrt3 + \sqrt{4c^2-a^2}}{2} \Rightarrow S = \frac12 ba\sin 30° = \frac{ab}{4}$

$\Rightarrow S = \overset{2}{\underline{a(a\sqrt3+\sqrt{4c^2-a^2})}} \Rightarrow a = 4\log_3^2$

$\Rightarrow 4c^2 - a^2 = \frac{36}{a^2} - 16a^2 = \frac{4(9-4a^4)}{a^2} \times \sqrt{4c^2-a^2} = \frac{(2\sqrt{9-4a^4}}{a} \Rightarrow S = 2\sqrt3(\log_3^2)^2 + \sqrt{9-4\log_3^2}$

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۵)

$2\sin 4 - \cancel{4}\sin 2 - \sin^2(\alpha)(2\sin^2\alpha - 1) \xrightarrow{\text{به ازای } \cos\alpha} 2\cos^4 - \cancel{4}\cos^2 - 2\sin^4\alpha + \sin^2\alpha = 1$

$\Rightarrow 2(\underbrace{\cos^4\alpha - \sin^4\alpha}) + \sin^2\alpha - \cos^2\alpha = 1 \Big\}\; 2\cos^2\alpha - 2\sin^2\alpha + \sin^2\alpha - \cos^2\alpha = 1$

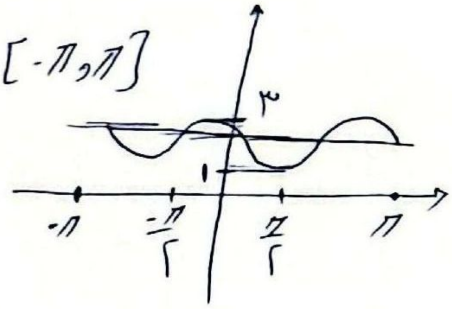
$(\underbrace{\cos^2\alpha + \cancel{\sin^2\alpha}}_{1})(\cos^2\alpha - \sin^2\alpha)$

$\Rightarrow \cos^2\alpha - \sin^2\alpha = 1 \Rightarrow \cos 2\alpha = 1 \Rightarrow \sin 2\alpha = 1 - \underbrace{\cos^2 2\alpha}_{1} = 0$

۶)

الف) $-2\underbrace{\sin x\cos x}_{\frac{1}{2}\sin 2x}+2$ $\quad [-\pi,\pi]$

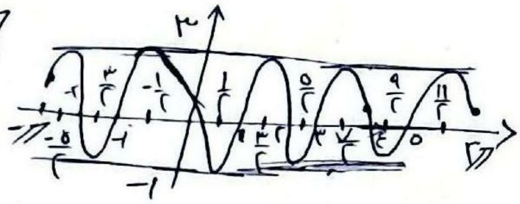
$T=\frac{2\pi}{2}=\pi$



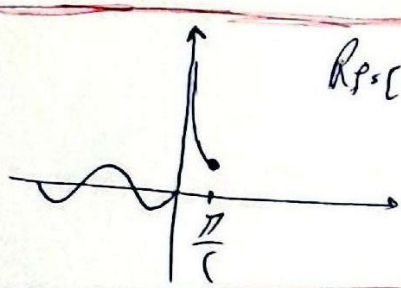
ب) $2\cos(\underbrace{x+\frac{1}{2}}_{2})\pi+1$ $\quad [-\pi, 2\pi]$

Sin تغییر نماد $\Leftarrow$ زاویه متمم $\frac{\pi}{x}+\frac{\pi}{2}$ $\qquad T=\frac{2\pi}{\pi}=2$

$-2\sin\pi x+1$



۷)



$R_f=[-1,+\infty)$ $\qquad f(x)=\begin{cases}\cot x\ ; & 0<x<\frac{\pi}{4}\\ \sin x\ ; & x\leq 0\end{cases}$

۸)

$a+b\sin(cx)\cos(cx)\cos(2cx)=a+\frac{b}{2}\sin(2cx)\cos(2cx)=a+\frac{b}{4}\sin 4cx$

$\Rightarrow f(x)=a+\frac{b}{4}\sin(4cx)\Rightarrow T=\frac{5\pi}{10}-\frac{\pi}{10}=\frac{4\pi}{10}=\frac{2\pi}{\omega}=\frac{2\pi}{4c}\Rightarrow \omega=4c\Rightarrow c=\frac{\omega}{4}$

$\left.\begin{array}{l}a+\frac{b}{4}=4\\ a-\frac{b}{4}=0\end{array}\right\}\xrightarrow{+}2a=4\Rightarrow a=2,\ 2+\frac{b}{4}=4\Rightarrow \frac{b}{4}=2\Rightarrow b=8\Rightarrow \frac{bc}{a}=\frac{\frac{5}{4}\times 8}{2}=5$

۹)

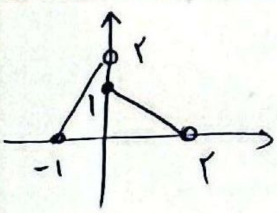
$f(x)=a\cos bx+c$ $\quad$ $a<0$ و $b>0$ $\Leftarrow$ نمودار cos ابتدا شیب مثبت دارد $\Leftarrow$ a و b مختلف علامت ، $\Rightarrow T=\frac{2\pi}{b}=\pi$

$\Rightarrow \underline{b=2}$ ، $\left.\begin{array}{l}max=-a+c=2\\ min=a+c=-4\end{array}\right\}\xrightarrow{+}2c=-2\Rightarrow \underline{c=-1}$ ، $a-1=-4\Rightarrow \underline{a=-3}$

$f(x)=-3\cos 2x-1\underset{\text{ریشه دارای}}{\Rightarrow}-3\cos 2x-1=0\Rightarrow \cos 2x=-\frac{1}{3}$ ، $1+\tan^2 2x=\frac{1}{\cos^2 2x}\Rightarrow 1+\tan^2 x=9$

$\Rightarrow \tan^2 2x=8\Rightarrow \tan 2x=\sqrt{8}$

۱۰)



$T=3\Rightarrow f(x)=f(x+3k)\Rightarrow f(-34.5)=f(1.5-36)$

$\Rightarrow f(\underbrace{1.5-3(12)}_{-34.5})=f(1.5)$ $\qquad f(x)=-\frac{1}{2}x+1\rightarrow f(1.5)=-0.75+1=\underline{0.25}$