۱- $f(x)=\tan(-2x+\pi/4) \to f(0)=\tan(\pi/4)=1 \Rightarrow B\Big|^{0}_{1}$

اولین نقاط تعریف نشده مثبت و منفی $\tan(\pi/2)$ و $\tan(-\pi/2) \Rightarrow -2x+\pi/4=\pm\pi/2 \Rightarrow x=\begin{cases}-\pi/8\\ 3\pi/8\end{cases}$

$\Rightarrow A\Big|^{-\pi/8}_{0} \;,\; C\Big|^{3\pi/8}_{0} \Rightarrow S_{ABC}=\frac{1}{2}(\pi/2)(1)=\pi/4$

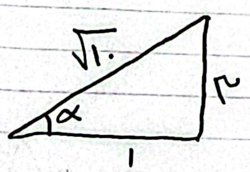
۲- $x=\frac{2}{3} \to \frac{16-24\sin-9\cos^2}{36} \xrightarrow{\cos^2=1-\sin^2} \frac{9\sin^2-24\sin+7}{36}=0 \overset{36\neq 0}{\Rightarrow}$

$9\sin^2-24\sin+7=0=(3\sin-1)(3\sin-7) \Rightarrow \sin\begin{cases}7/3 \;✗\\ 1/3 \;✓\end{cases}$

$\sin=1/3 \Rightarrow \cos^2=1-1/9=8/9 \Rightarrow 1+\tan^2=\frac{1}{\cos^2}=9/8$

۳- $\log\left(\frac{\sin}{-\cos}\right)=\log(-\tan\alpha)=\log 3 \Rightarrow \tan\alpha=-3$

log آرگومان $>0 = D \Rightarrow \sin>0 \;,\; \cos<0 \Rightarrow \alpha \to$ ناحیه دوم



$\sin\alpha=\frac{3}{\sqrt{10}} \;,\; \cos=-\frac{1}{\sqrt{10}} \Big\} \Rightarrow \sin-\cos=\frac{3-(-1)}{\sqrt{10}}=\frac{4}{\sqrt{10}}=\frac{2\sqrt{10}}{5}$

۴- $\cos^2=1-\sin^2$ $\xrightarrow{\text{صورت سوال}}$ $\sin^4-\sin^2+1/5=0 \xrightarrow{\sin^2\alpha=r}$

$r^2-r+1/5=0$ جمع ریشه‌ها $=\frac{-(-1)}{1}=1 \Rightarrow \sin^2\alpha+\sin^2\beta=1 \Rightarrow$ ریشه‌ها متمم یکدیگرند

$\Rightarrow \sin^4\alpha=\cos^4\beta \;,\; \sin^2\alpha=\cos^2\beta \Rightarrow \cos^4\theta+\sin^2\theta=\sin^4\theta+\cos^2\theta=4/5 \Rightarrow$

$\cos^4\theta+\sin^2\theta=4/5$

۵-

$$\frac{3\sin+\sin\cos}{2-2\cos^2}=\frac{\sin(3+\cos)}{2(1-\cos^2)}<0 \Rightarrow \sin x<0$$

$$\sin\cdot\tan-\frac{1}{\cos}=\frac{\sin^2-1}{\cos}=\frac{-\cos^2}{\cos}>0 \Rightarrow \cos x<0$$

⇐ در ناحیه ۳

۶-

$$\frac{2}{\sin}+\frac{3}{\cos}=\frac{2\cos+3\sin}{\sin\cos}=0 \Rightarrow \sin=-\frac{2}{3}\cos \Rightarrow \tan x=-\frac{2}{3}\ ,$$

$$\cot x=-\frac{3}{2} \Rightarrow \tan x+\cot x=-\frac{2}{3}-\frac{3}{2}=\frac{-13}{6}$$

۷- تبدیل جمع به ضرب ← $\sin 50^\circ+\sin 10^\circ=2\sin(30^\circ)\cdot\cos(20^\circ)$

مخرج ← $\cos 20^\circ$ (I) / صورت ⇒ $1+\sin 50^\circ=1+\cos 40^\circ=1+(2\cos^2 20^\circ-1)=$

← $2\cos^2 20^\circ$ (II) $\xrightarrow{\text{(I)},\ \text{(II)}}$ $\dfrac{\sqrt{2\cos^2 20^\circ}}{\cos 20^\circ}=\sqrt{2}$

۸- از طرفی: $\cos 2x=2\cos^2-1=1-2\sin^2=\dfrac{1-\tan^2}{1+\tan^2} \Rightarrow$

$$\cos 10^\circ\times\cos 20^\circ\times\cos 40^\circ\times\frac{\sin 10^\circ}{\sin 10^\circ}=\frac{1}{8}\,\frac{\sin 80^\circ}{\sin 10^\circ}=\frac{\cot 10^\circ}{8}=\frac{\tan 80^\circ}{8}$$

۷- راه ۲:

$$\frac{\sqrt{1+\cos 40^\circ}}{\sin(20^\circ+30^\circ)+\sin(30^\circ-20^\circ)}=\frac{\sqrt{2\cos^2 20^\circ}}{2\sin 30^\circ\cdot\cos 20^\circ}=\frac{\sqrt{2}\cos 20^\circ}{\cos 20^\circ}=\sqrt{2}$$

$$\sin x + 3\cos x = 2 \Rightarrow \sin x = 2 - 3\cos x \Rightarrow \sin^2 x = 4 - 12\cos x + 9\cos^2 x \Rightarrow$$ ۹-

$$\Rightarrow 10\cos^2 x - 12\cos x + 3 = 0 \Rightarrow 10\cos^2 x - 12\cos x = -3 \quad \mathrm{I}$$

خواسته سوال: $5\cos 2x - 12\cos x = 10\cos^2 x - 12\cos x - 5 \xrightarrow{\mathrm{I}}$

$$-3 - 5 = \boxed{-8}$$

$$\sin B \cdot \sin C = \cos^2\frac{A}{2} = \frac{1+\cos\hat{A}}{2} \Rightarrow 2\sin B \cdot \sin C = 1 + \cos\hat{A} \Longrightarrow$$ ۱۰-

$$\xrightarrow{\hat{A} = \pi - (B+C)} 2\sin B \sin C = 1 + \cos(\pi - (B+C)) = 1 - \cos(B+C) \Rightarrow$$

$$2\sin B \cdot \sin C = 1 - \cos B \cdot \cos C + \sin B \cdot \sin C \Rightarrow \sin B \cdot \sin C + \cos B \cdot \cos C = 1 \Longrightarrow$$

$$\cos(B - C) = 1 = \cos(0) \Rightarrow B - C = 0 \Rightarrow \hat{B} = \hat{C} = 72 \Rightarrow \boxed{\hat{A} = 36^\circ}$$