نام و نام خانوادگی ...امیرعلی کیوان نژاد... پاسخنامه تشریحی تکلیف شماره ...۱۵... کلاس ...دوازدهم A...

۱۸

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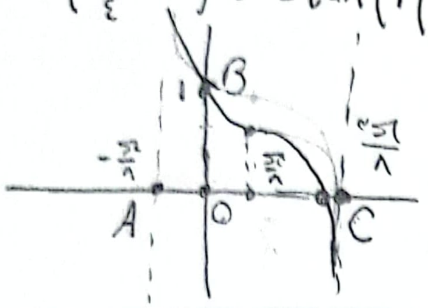
$\tan\left(\frac{\pi}{4}-2x\right)=-\tan\left(2\left(x-\frac{\pi}{8}\right)\right)$ $T=\frac{\pi}{2}$



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$AC=\frac{\pi}{2}$ $B=f(0)=\tan\frac{\pi}{4}=1$

$\Rightarrow S_{ABC}=\frac{AC\times BO}{2}=\frac{\pi}{2}\times1\times\frac{1}{2}=\frac{\pi}{4}$

۲

$\frac{4}{9}-\frac{2}{3}\sin\alpha-\frac{1}{4}\cos^2\alpha=0$ ($\cos^2\alpha = 1-\sin^2\alpha$)

$\Rightarrow \frac{4}{9}-\frac{2}{3}\sin\alpha-\frac{1}{4}(1-\sin^2\alpha)=0$

$\Rightarrow 16-24\sin\alpha-9(1-\sin^2\alpha)=9\sin^2\alpha-24\sin\alpha+7$

$\sin\alpha=\frac{24\pm18}{18}\Rightarrow\sin\alpha=\frac{1}{3}$

3, 1, $\sqrt{8}$

$1+\tan^2\alpha=1+\frac{1}{8}=\boxed{\frac{9}{8}}$

۲

۳

$\log^{-\frac{\sin\alpha}{\cos\alpha}}=\log^{3}\Rightarrow-\tan\alpha=3\Rightarrow\tan\alpha=-3$ $\sin\alpha>0$ $\cos\alpha<0$

۲

$\sqrt{10}$, 3, 1

$\Rightarrow \left.\begin{array}{l}\cos\alpha=-\frac{1}{\sqrt{10}}\\ \sin\alpha=\frac{3}{\sqrt{10}}\end{array}\right\}\Rightarrow\sin\alpha-\cos\alpha=\frac{4}{\sqrt{10}}=\frac{2\sqrt{10}}{5}$

۵

$\frac{\sin\alpha(3+\cos\alpha)}{2(1-\cos^2\alpha)}=\frac{\sin\alpha(3+\cos\alpha)}{2\sin^2\alpha}=\frac{3+\cos\alpha}{2\sin\alpha}<0\xrightarrow[\text{صورت همواره بزرگتر از صفر است}]{}\sin\alpha<0$ (1)

همواره کوچک‌تر از صفر

۲

$\sin x\tan x-\frac{1}{\cos x}=\frac{\sin^2x-1}{\cos x}>0\Rightarrow\cos x<0$ (2)

(1), (2) $\Rightarrow \pi<\alpha<\frac{3\pi}{2}$

در ناحیه سوم

۴

$\sin^4\alpha+1-\sin^2\alpha=\frac{4}{5}\Rightarrow \sin^4\alpha-\cancel{\sin^2\alpha}=-\frac{1}{5}$

$\cos^4\alpha+\sin^4\alpha+2\sin^2\alpha\cancel{\cos^2\alpha}=1$ ($1-\sin^2\alpha$; $2\sin^2\alpha-2\sin^4\alpha$) $\Big\}\Rightarrow\cos^4\alpha=\sin^4\alpha-2\sin^2\alpha\Rightarrow\cos^4\alpha+\sin^2\alpha=\sin^4\alpha-\sin^2\alpha$ $\boxed{-\frac{1}{5}}$

$(\sin^2\theta)^2+\cos^2\theta=\frac{4}{5}$

$\Rightarrow(1-\cos^2\theta)^2+\cos^2\theta=\frac{4}{5}$

$\to\cos^4\theta-\cos^2\theta=-\frac{1}{5}$

$\cos^4\theta-(1-\sin^2\theta)=-\frac{1}{5}$

$\cos^4\theta+\sin^2\theta=\frac{4}{5}$

$2\cos x + 3\sin x = 0 \Rightarrow 3\sin x = -2\cos x \Rightarrow \frac{\sin x}{\cos x} = -\frac{2}{3} = \tan x \Rightarrow \cot x = -\frac{3}{2}$

$\Rightarrow -\frac{2}{3} - \frac{3}{2} = \frac{-4-9}{6} = -\frac{13}{6}$

2 ✓ ۶

$\frac{1+\sin 50}{\sin 10 + \sin 50} = \frac{\sin 25 + \cos 25}{\sin(30-20) + \sin(30+20)} = \frac{\sin 25 + \cos 25}{\sin 30\cos 20 - \cos 30\sin 20 + \sin 30\cos 20 + \cos 30\sin 20}$

$= \frac{\sin 25 + \cos 25}{2\underbrace{\sin 30}_{\frac{1}{2}}\cos 20} = \frac{\sqrt{2}\sin(25+45)}{\cos 20} = \frac{\sqrt{2}\sin 70}{\cos 20} = \boxed{\sqrt{2}}$

2 ✓ ۷

$(\cos 10)(\cos 20)(\cos 40) = \frac{\sin(10)\cos(10)\cos(20)(\cos 40)}{\sin 10} = \frac{\sin 80}{8\sin 10} = \boxed{\frac{1}{8}\cot 80^\circ}$

$1 - 2\sin^2 5 = \cos 10$

$2\cos^2 10 - 1 = \cos 20$

$\frac{1-\tan^2 20}{1+\tan^2 20} = \cos 40$

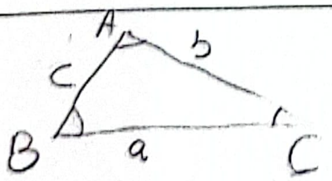
2 ✓ ۸

$5(2\cos^2\alpha - 1) - 12\cos\alpha = 10\cos^2\alpha - 12\cos\alpha - 5$

$(\sin\alpha)^2 = (2 - 3\cos\alpha)^2 \Rightarrow \underbrace{\sin^2\alpha}_{1-\cos^2\alpha} = 4 - 12\cos\alpha + 9\cos^2\alpha \Rightarrow 10\cos^2\alpha + 3 - 12\cos\alpha = 0$

$\Rightarrow 10\cos^2\alpha - 12\cos\alpha - 5 = \boxed{-8}$

2 ✓ ۹



$\cos^2\frac{A}{2} = \frac{1+\cos A}{2} \Rightarrow \cos A = \cos(\pi - (B+C)) = -\cos(B+C)$

$1 - \cos(B+C) = 2\sin B\sin C$ ۱۰

$\Rightarrow 1 - (\cos B\cos C - \sin B\sin C) = 2\sin B\sin C$

$1 - (\cos B\cos C + \sin B\sin C) = 0 \Rightarrow 1 - \cos(B-C) = 0 \Rightarrow \cos(B-C) = 1$

$\Rightarrow B - C = 0$

2 ✓

$\Rightarrow B = 72,\ C = 72 \Rightarrow A = 36$