آرمین نوری

۱۸/۵

۱-

$T = \frac{\pi}{|-2|} = \pi/2 = AC$

$B \Rightarrow x = 0 \to B = \tan \pi/4 = 1$

$S_{\text{مثلث}} = \frac{B \times AC}{2} = \frac{\pi/2}{2} = \pi/4$

۲ ✓

۲-

$-\cos^2 x = \frac{1}{4}\sin^2 x - \frac{1}{4}$ $\quad x^2 - (\sin\alpha)x + \frac{1}{4}\sin^2\alpha - \frac{1}{4} = 0$

$\frac{4}{9} - \frac{2}{3}\sin\alpha + \frac{1}{4}\sin^2\alpha - \frac{1}{4} = 0 \longrightarrow 9\sin^2\alpha - 24\sin\alpha + 7 = 0$ $\quad \sin^2\alpha - 24\sin\alpha + 63$

$(\sin\alpha - 21)(\sin\alpha - 3) = 0$ $\quad \sin\alpha < \begin{cases} 21/9 = 7/3 \ \times \\ 3/9 = 1/3 \ ✓ \end{cases}$

$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} = \frac{1}{\frac{8}{9}} = 9/8$

۲ ✓

۳-

$\log(\sin\alpha) - \log(-\cos\alpha) = \log\frac{\sin\alpha}{-\cos\alpha} = \log 3$ $\quad \frac{\sin\alpha}{-\cos\alpha} = 3$ $\quad -\tan\alpha = 3$

$\sin\alpha = \frac{3}{\sqrt{10}}$ $\quad \cos\alpha = -\frac{1}{\sqrt{10}}$ $\quad \sin\alpha - \cos\alpha = \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{10}} = \frac{4}{\sqrt{10}} = \frac{4\sqrt{10}}{10}$

۲ ✓

۴-

$\sin^4\theta = (1-\cos^2\theta)^2 = 1 + \cos^4\theta - 2\cos^2\theta$

$\cos^4\theta - 2\cos^2\theta + 1 + \cos^2\theta = \cos^4\theta - \cos^2\theta + 1 = \frac{4}{5}$

$\cos^4\theta + \sin^2\theta = \cos^4\theta - \cos^2\theta + 1 = 4/5$

۲ ✓

۵-

$\frac{\sin^2 x}{\cos x} - \frac{1}{\cos x} > 0$ $\quad \frac{\sin^2 x - 1}{\cos x} > 0$ $\quad (-1, 0] \Longrightarrow \cos x < 0$

$\frac{3\sin x + \sin x\cos x}{2 - 2\cos^2 x} = \frac{\sin x(3 + \cos x)}{2\sin^2 x} < 0 \Rightarrow \sin x < 0$

ناحیه‌ی سوم

۲ ✓

آزمون تئوری

۶- $\frac{2}{\sin x} + \frac{3}{\cos x} = 0 \rightarrow \frac{2\cos x + 3\sin x}{\sin x \cos x} = 0 \qquad 2\cos x + 3\sin x = 0$

$2\cos x = -3\sin x \qquad \tan x = -\frac{2}{3} \qquad \cot x = -\frac{3}{2} \qquad \tan x + \cot x = \frac{-4-9}{6} = \frac{-13}{6}$

②

۷- $1+\sin 50 = 1+\cos 40 = \sqrt{2\cos^2 20} = \sqrt{2}\cos 20$

$\sin 50 + \sin 10 = \sin(30+20) + \sin(30-20)$

$\frac{\sqrt{2}\cos 20}{2\sin 30 \cos 20} = \sqrt{2}$

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۸- $1-2\sin^2 5 = \sin^2 5 + \cos^2 5 - 2\sin^2 5 = \cos^2 5 - \sin^2 5 = \cos 10 \qquad \frac{1-\tan^2 20}{1+\tan^2 20} = \cos 40$

$2\cos^2 10 - 1 = 2\cos^2 10 - \sin^2 10 - \cos^2 10 = \cos^2 10 - \sin^2 10 = \cos 20$

$\cos 10 \times \cos 20 \times \cos 40 \xrightarrow{\times \frac{\sin 10}{\sin 10}} \frac{1}{\sin 10}(\sin 10 \cos 10 \times \cos 20 \times \cos 40) = \frac{1}{\sin 10}(\frac{1}{2}\sin 20 \cos 20 \times \cos 40)$

$\frac{1}{\sin 10}(\frac{1}{4}\sin 40 \cos 40) = \frac{1}{\sin 10} \times \frac{1}{8}\sin 80 = \frac{1}{8}\frac{\cos 10}{\sin 10} = \frac{1}{8}\cot 10$

②

۹- $\sin\alpha + 3\cos\alpha = 2 \rightarrow \sin\alpha = 2 - 3\cos\alpha \qquad \cos^2\alpha + (2-3\cos\alpha)^2 = 1$

$\cos^2\alpha + 4 + 9\cos^2\alpha - 12\cos\alpha = 1 \qquad 10\cos^2\alpha - 12\cos\alpha = -3 \qquad \cos 2\alpha = 2\cos^2\alpha - 1$

$5\cos 2\alpha = 10\cos^2\alpha - 5 \qquad 10\cos^2\alpha - 12\cos\alpha - 5 = -8$

②

۱۰- $\sin a \times \sin b = \frac{1}{2}(\cos(a-b) - \cos(a+b)) \rightarrow \sin B \sin C = \frac{1}{2}(\cos(B-C) - \cos(B+C))$

$\cos a \times \cos b = \frac{1}{2}(\cos(a+b) + \cos(a-b)) \rightarrow \cos\frac{A}{2} \times \cos\frac{A}{2} = \frac{1}{2}(\cos A + \cos)$

$A+B+C = 180 \rightarrow B+C = 180 - A, \quad B = 180 - C - A \Rightarrow \frac{1}{2}(\cos(180-(2C+A)) - \cos(180-A))$

$= \frac{1}{2}(\cos 2C+A + \cos A)$

$\frac{1}{2}\cos A + \frac{1}{2} = -\frac{1}{2}\cos 2C+A + \frac{1}{2}\cos A \rightarrow -\frac{1}{2}\cos 2C+A = \frac{1}{2} \qquad \cos 2C+A = -1$

$2C+A = 180 \qquad C = B = 72 \qquad A = 36$

②