آرمین نوری

۱-

$T = \frac{\pi}{|-۲|} = \pi/۲ = AC$

$B \Rightarrow x = ۰ \rightarrow B = \tan \pi/۴ = ۱$

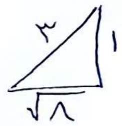
$S_{\text{مثلث}} = \frac{B \times AC}{۲} = \frac{\pi/۲}{۲} = \boxed{\pi/۴}$

۲-

$-\cos^۲ x = \frac{۱}{۴}\sin^۲ x - \frac{۱}{۴}$ $\qquad x^۲ - (\sin\alpha)x + \frac{۱}{۴}\sin^۲\alpha - \frac{۱}{۴} = ۰$

$\frac{۴}{۹} - \frac{۲}{۳}\sin\alpha + \frac{۱}{۴}\sin^۲\alpha - \frac{۱}{۴} = ۰ \longrightarrow ۹\sin^۲\alpha - ۲۴\sin\alpha + ۷ = ۰$ $\qquad \sin^۲\alpha - ۲۴\sin\alpha + ۶۳$

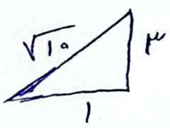
$(\sin\alpha - ۲۱)(\sin\alpha - ۳) = ۰$ $\qquad \sin\alpha < \begin{cases} ۲۱/۹ = ۷/۳ \times \\ ۳/۹ = ۱/۳ \checkmark \end{cases}$



$1 + \tan^۲\alpha = \frac{۱}{\cos^۲\alpha} = \frac{۱}{\frac{۸}{۹}} = \boxed{۹/۸}$

۳-

$\log(\sin\alpha) - \log(-\cos\alpha) = \log\frac{\sin\alpha}{-\cos\alpha} = \log ۳$ $\qquad \frac{\sin\alpha}{-\cos\alpha} = ۳$ $\qquad -\tan\alpha = ۳$



$\sin\alpha = \frac{۳}{\sqrt{۱۰}}$ $\qquad \cos\alpha = -\frac{۱}{\sqrt{۱۰}}$ $\qquad \sin\alpha - \cos\alpha = \frac{۳}{\sqrt{۱۰}} + \frac{۱}{\sqrt{۱۰}} = \frac{۴}{\sqrt{۱۰}} = \boxed{\frac{۴\sqrt{۱۰}}{۱۰}}$

۴-

$\sin^۴\theta = (1 - \cos^۲\theta)^۲ = 1 + \cos^۴\theta - ۲\cos^۲\theta$

$\cos^۴\theta - ۲\cos^۲\theta + ۱ + \cos^۲\theta = \cos^۴\theta - \cos^۲\theta + ۱ = \frac{۴}{۵}$

$\cos^۴\theta + \sin^۲\theta = \cos^۴\theta - \cos^۲\theta + ۱ = \boxed{۴/۵}$

۵-

$\frac{\sin^۲ x}{\cos x} - \frac{۱}{\cos x} > ۰$ $\qquad \frac{\sin^۲ x - ۱}{\cos x} > ۰ \qquad (-۱, ۰] \Longrightarrow \cos x < ۰$

$\frac{۳\sin x + \sin x\cos x}{۲ - ۲\cos^۲ x} = \frac{\sin x\,\overbrace{(۳ + \cos x)}^{+}}{\underbrace{۲\sin^۲ x}_{+}} < ۰ \Rightarrow \sin x < ۰$

$\boxed{\text{ناحیه‌ی سوم}}$

آرمین نوری

۶-

$$\frac{2}{\sin x}+\frac{3}{\cos x}=0 \longrightarrow \frac{2\cos x+3\sin x}{\sin x\cos x}=0 \qquad 2\cos x+3\sin x=0$$

$$2\cos x=-3\sin x \qquad \tan x=-\frac{2}{3} \qquad \cot x=-\frac{3}{2} \qquad \tan x+\cot x=\frac{-4-9}{6}=\frac{-13}{6}$$

۷-

$$1+\sin 50=1+\cos 40=2\cos^2 20$$

۸-

$$1-2\sin^2 5=\sin^2 5+\cos^2 5-2\sin^2 5=\cos^2 5-\sin^2 5=\cos 10 \qquad \frac{1-\tan^2 20}{1+\tan^2 20}=\cos 40$$

$$2\cos^2 10-1=2\cos^2 10-\sin^2 10-\cos^2 10=\cos^2 10-\sin^2 10=\cos 20$$

$$\cos 10\times\cos 20\times\cos 40 \xrightarrow{\times\frac{\sin 10}{\sin 10}} \frac{1}{\sin 10}\left(\sin 10\cos 10\times\cos 20\times\cos 40\right)=\frac{1}{\sin 10}\left(\frac{1}{2}\sin 20\cos 20\times\cos 40\right)$$

$$\frac{1}{\sin 10}\left(\frac{1}{4}\sin 40\cos 40\right)=\frac{1}{\sin 10}\times\frac{1}{8}\sin 80=\frac{1}{8}\,\frac{\cos 10}{\sin 10}=\frac{1}{8}\cot 10$$

۹-

$$\sin\alpha+3\cos\alpha=2 \longrightarrow \sin\alpha=2-3\cos\alpha \qquad \cos^2\alpha+(2-3\cos\alpha)^2=1$$

$$\cos^2\alpha+4+9\cos^2\alpha-12\cos\alpha=1 \qquad 10\cos^2\alpha-12\cos\alpha=-3 \qquad \cos 2\alpha=2\cos^2\alpha-1$$

$$5\cos 2\alpha=10\cos^2\alpha-5 \qquad 10\cos^2\alpha-12\cos\alpha-5=-8$$

۱۰-

$$\sin a\times\sin b=\frac{1}{2}\left(\cos(a-b)-\cos(a+b)\right) \longrightarrow \sin B\sin C=\frac{1}{2}\left(\cos(B-C)-\cos(B+C)\right)$$

$$\cos a\times\cos b=\frac{1}{2}\left(\cos(a+b)+\cos(a-b)\right) \longrightarrow \cos\frac{A}{2}\times\cos\frac{A}{2}=\frac{1}{2}\left(\cos A+\cos\right)$$

$$A+B+C=180 \longrightarrow B+C=180-A,\quad B=180-C-A \implies \frac{1}{2}\left(\cos(180-(2C+A))-\cos(180-A)\right)$$

$$=\frac{1}{2}\left(\cos 2C+A+\cos A\right)$$

$$\frac{1}{2}\cos A+\frac{1}{2}=-\frac{1}{2}\cos 2C+A+\frac{1}{2}\cos A \longrightarrow -\frac{1}{2}\cos 2C+A=\frac{1}{2} \qquad \cos 2C+A=-1$$

$$2C+A=180 \qquad C=B=72 \qquad A=36$$