(7)

$$\frac{\sqrt{1+\sin 50^\circ}}{\underbrace{\sin 50^\circ}_{\sin(30^\circ+20^\circ)}+\underbrace{\sin 10^\circ}_{\sin(30^\circ-20^\circ)}}=\frac{\sqrt{(\sin 25^\circ+\cos 25^\circ)^2}}{\sin 20^\circ\cos 30^\circ+\sin 30^\circ\cos 20^\circ+\sin 30^\circ\cos 20^\circ-\sin 20^\circ\cos 30^\circ}=\frac{\sin 25^\circ+\cos 25^\circ}{2\underbrace{\sin 30^\circ}_{\frac{1}{2}}\cos 20^\circ}$$

$$=\frac{\sqrt{2}\sin(25^\circ+45^\circ)}{\cos 20^\circ}=\frac{\sqrt{2}\sin 70^\circ}{\cos 20^\circ}=\sqrt{2}$$

(8)

$$\Big(\underbrace{1-2\sin^2 5^\circ}_{\cos 10^\circ}\Big)\Big(\underbrace{2\cos^2 10^\circ-1}_{\cos 20^\circ}\Big)\Big(\underbrace{\frac{1-\tan^2 20^\circ}{1+\tan^2 20^\circ}}_{\cos 40^\circ}\Big)=\cos 10^\circ\cos 20^\circ\cos 40^\circ \xrightarrow{\times\frac{\sin 10^\circ}{\sin 10^\circ}} \frac{\overbrace{\sin 10^\circ\cos 10^\circ}^{\frac{1}{2}\sin 20^\circ}\cos 20^\circ\cos 40^\circ}{\sin 10^\circ}$$

$-2+2\cos^2 \alpha$

$$=\frac{1}{2}\,\frac{\overbrace{\sin 20^\circ\cos 20^\circ}^{\frac{1}{2}\sin 40^\circ}\cos 40^\circ}{\sin 10^\circ}\longrightarrow \frac{1}{2}\times\frac{1}{2}\times\frac{\overbrace{\sin 40^\circ\cos 40^\circ}^{\frac{1}{2}\sin 80^\circ}}{\sin 10^\circ}=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{\overbrace{\sin 80^\circ}^{\cos 10^\circ}}{\sin 10^\circ}=\frac{1}{8}\cot 10^\circ$$

(9)

$$\sin x+3\cos x=2 \xrightarrow{(1)} \sin x=2-3\cos x \xrightarrow{\text{به توان ۲}} \underbrace{\sin^2 x}_{1-\cos^2 x}=4-12\cos x+9\cos^2 x$$

$$\Rightarrow 10\cos^2 x+3-12\cos x=0$$

$$(2)\ 5\cos 2x-12\cos x=5(2\cos^2 x-1)-12\cos x=10\cos^2 x-5-12\cos x$$

$$10\cos^2 x+3-12\cos x=0 \xrightarrow{-8} 10\cos^2 x-5-12\cos x=\boxed{-8}$$ ⇐ عبارت ① و ② که یکی از آنها جواب است ، ۸ تا اختلاف دارند

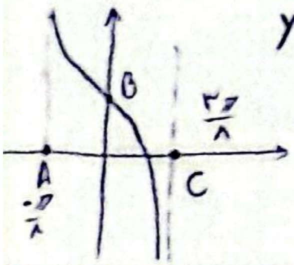
(10)

$$\sin \hat{B}\sin \hat{C}=\cos^2\frac{\hat{A}}{2},\quad \hat{B}=72^\circ$$

$$\hookrightarrow \sin \hat{B}\sin \hat{C}=\frac{1+\cos A}{2}$$

آرا آزادنیا — تکلیف ۱۵ — دوازدهم میان A

①



$y=\tan(-2x+\frac{\pi}{4})=\tan(2(\frac{\pi}{8}-x))\Rightarrow T=\frac{4\pi}{8}$

$AC=\frac{\pi}{2}$, $B=f(0)=\tan\frac{\pi}{4}=1$, $S_{ABC}=\frac{1}{2}AC\times BO=\frac{\pi}{2}\times1\times\frac{1}{2}=\frac{\pi}{4}$

---

②

$x^2-(\sin\alpha)x-\frac{1}{4}\cos^2\alpha=0$, $f(\frac{2}{3})=0\Rightarrow\frac{4}{9}-\frac{2}{3}\sin\alpha-\frac{1}{4}\cos^2\alpha=0$

$\Rightarrow16-24\sin\alpha-9(1-\sin^2\alpha)=0\Rightarrow9\sin^2\alpha-24\sin\alpha+7=0\Rightarrow\sin\alpha=\frac{24\pm\sqrt{324}}{18}$ ⟨ $\frac{42}{18}$ ✗ , $\frac{6}{18}$ ✓

$\Rightarrow\cos^2\alpha=1-\sin^2\alpha=\frac{8}{9}\Rightarrow1+\tan^2\alpha=\frac{1}{\cos^2\alpha}=\frac{1}{\frac{8}{9}}=\frac{9}{8}$

---

③

$\log(\sin\alpha)-\log(-\cos\alpha)=\log3\Rightarrow\log\frac{\sin}{\cos}=\log3\Rightarrow-\tan\alpha=3$, (مثلث با اضلاع $\sqrt{10}$ ، ۳ ، ۱) $\tan=\frac{\text{مقابل}}{\text{مجاور}}=\frac{3}{1}$

$\Rightarrow\cos\alpha=\frac{-1}{\sqrt{10}}=\frac{-\sqrt{10}}{10}$, $\sin\alpha=\frac{3}{\sqrt{10}}=\frac{3\sqrt{10}}{10}\Rightarrow\sin\alpha-\cos\alpha=\frac{3\sqrt{10}}{10}+\frac{\sqrt{10}}{10}=\frac{4\sqrt{10}}{10}=\frac{2\sqrt{10}}{5}$

---

④

$\sin^4\theta+\underbrace{\cos^2\theta}_{1-\sin^2}=\frac{4}{5}\xrightarrow{①}\sin^4\theta+1-\sin^2\theta=\frac{4}{5}\Rightarrow\sin^4\theta-\sin^2\theta=\frac{-1}{5}$

$\underbrace{(\sin^2\theta+\cos^2\theta)^2}_{1}=\sin^4+\underbrace{2\sin^2\theta\cos^2\theta}_{2\sin^2\theta-2\sin^4\theta}+\cos^4\theta=1\Rightarrow\cos^4\theta+\sin^2\theta=1+\underbrace{\sin^4\theta-\sin^2\theta}_{\frac{-1}{5}}$

$\Rightarrow\cos^4\theta+\sin^2\theta=\frac{4}{5}$

---

⑤

$\frac{3\sin x+\sin x\cos x}{2-2\cos^2x}>0$ و $\sin x\tan x-\frac{1}{\cos x}>0\xrightarrow{②}\sin x\times\frac{\sin x}{\cos x}-\frac{1}{\cos x}=\frac{\sin^2x-1}{\cos x}>0\Rightarrow\cos x<0$

صورت همواره منفی

① $\frac{\sin x(3+\cos x)}{2\underbrace{(1-\cos^2x)}_{\sin^2x}}=\frac{3+\cos x}{2\sin x}\Rightarrow\sin x>0$

صورت همواره مثبت

$\sin x>0$, $\cos x<0$ } x ناحیه دوم

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⑥

$\frac{2}{\sin x}+\frac{3}{\cos x}=0\Rightarrow\frac{2\cos x+3\sin x}{\sin x\cos x}=0\xrightarrow{\text{صورت صفر}}2\cos x=-3\sin x\Rightarrow\frac{-2\cos x}{3\sin x}=1$

$\Rightarrow\tan x=\frac{-2}{3}\Rightarrow\cot x=\frac{-3}{2}\Rightarrow\tan x+\cot x=\frac{-2}{3}+\frac{-3}{2}=\frac{-4-9}{6}=\frac{-13}{6}$