نام و نام خانوادگی ...بردیا یلدا یان... پاسخنامه تشریحی تکلیف شماره ..۱۵.. کلاس ...چهارم...A

۱

$AC = \frac{\text{دوره}}{\text{تناوب}} \rightarrow T = \frac{\pi}{|-2|} = \frac{\pi}{2}$

$B \Rightarrow x = 0 \rightarrow B = \tan\frac{\pi}{4} = 1$

$S_{\text{مثلث}} = \frac{\text{ارتفاع} \times \text{قاعده}}{2} = \frac{AC \times B}{2} = \frac{\frac{\pi}{2} \times 1}{2} = \boxed{\frac{\pi}{4}}$

۲

$x^2 - \sin\alpha\, x - \frac{1}{4}\cos^2\alpha = 0 \xrightarrow{x = \frac{2}{3}} \frac{4}{9} - \frac{2}{3}\sin\alpha - \frac{1}{4}(1 - \sin^2\alpha) = 0$

$\rightarrow \frac{4}{9} - \frac{2}{3}\sin\alpha - \frac{1}{4} + \frac{1}{4}\sin^2\alpha = 0 \rightarrow \frac{1}{4}\sin^2\alpha - \frac{2}{3}\sin\alpha + \frac{7}{36} = 0$

$\xrightarrow{\times 36} 9\sin^2\alpha - 24\sin\alpha + 7 = 0 \rightarrow \Delta = (-24)^2 - 4 \times 7 \times 9 = 324$

$\rightarrow \sin\alpha = \frac{24 \pm \sqrt{324}}{18} = \frac{24 \pm 18}{18}$ → $\frac{7}{3}$ غ ق ق ← بزرگتر از یکه ؛ → $\frac{1}{3}$ ق ق

$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha}$

$\sin^2\alpha + \cos^2\alpha = 1 \rightarrow \frac{1}{9} + \cos^2\alpha = 1 \rightarrow \cos^2\alpha = \frac{8}{9}$

$= \frac{1}{\frac{8}{9}} = \boxed{\frac{9}{8}}$

۳

$\log^{\sin\alpha} - \log^{-\cos\alpha} = \log^{3} \rightarrow \log^{\frac{\sin\alpha}{-\cos\alpha}} = \log^{3} \rightarrow \frac{\sin\alpha}{-\cos\alpha} = 3 \rightarrow \sin\alpha = -3\cos\alpha$

$\sin^2\alpha + \cos^2\alpha = 1 \rightarrow (-3\cos\alpha)^2 + \cos^2\alpha = 1 \rightarrow 10\cos^2\alpha = 1 \rightarrow \cos^2\alpha = \frac{1}{10}$

$\rightarrow \cos\alpha =$ $\frac{1}{\sqrt{10}}$ × → زیرا جلوی لوگ را منفی می کند ؛ $-\frac{1}{\sqrt{10}}$ ✓

$\sin\alpha = -\frac{3}{-\sqrt{10}} = \frac{3}{\sqrt{10}}$

$\sin\alpha - \cos\alpha = \frac{3}{\sqrt{10}} - \left(-\frac{1}{\sqrt{10}}\right) = \frac{4}{\sqrt{10}} = \frac{4\sqrt{10}}{10} = \boxed{0/4\sqrt{10}}$

۴

$\sin^4\theta + \cos^2\theta = \frac{4}{5} \xrightarrow{\cos^2\theta = 1 - \sin^2\theta} \sin^4\theta - \sin^2\theta + \frac{1}{5} = 0$ (1)

$\sin^4\theta + \cos^2\theta = \frac{4}{5} \xrightarrow{\sin^2\theta = 1 - \cos^2\theta} (\sin^2\theta)^2 + \cos^2\theta = \frac{4}{5} \rightarrow (1 - \cos^2\theta)^2 + \cos^2\theta = \frac{4}{5}$

$1 + \cos^4\theta - 2\cos^2\theta + \cos^2\theta = \frac{4}{5} \rightarrow \cos^4\theta - \cos^2\theta + \frac{1}{5} = 0$ (2)

$(1) = (2) \rightarrow \sin^4\theta - \sin^2\theta = \cos^4\theta - \cos^2\theta \rightarrow \cos^4\theta + \sin^2\theta = \sin^4\theta + \cos^2\theta$

$= \boxed{\frac{4}{5}}$

۵

$\frac{\sin^2 x}{\cos x} - \frac{1}{\cos x} > 0 \rightarrow \frac{\sin^2 x - 1}{\cos x} > 0$ → منفی است همواره → $\cos x < 0$ (1)

$\frac{\sin x(3 + \cos x)}{2 - 2\cos^2 x} < 0 \rightarrow \frac{\sin x\,\overbrace{(3 + \cos x)}^{\text{همواره } +}}{\underbrace{2\sin^2 x}_{\text{همواره } +}} < 0 \rightarrow \sin x < 0$ (2)

$2 - 2\cos^2 x$
$\downarrow$
$2(1 - \cos^2 x)$
$2\sin^2 x$

$(1) \cap (2) \rightarrow$ ناحیه سوم

$\frac{2}{\sin x}+\frac{3}{\cos x}=0 \to \frac{2\cos x+3\sin x}{\sin x\cos x}=0 \to 2\cos x+3\sin x=0 \to 2\cos x=-3\sin x$

$\to \cos x=-\frac{3}{2}\sin x$

$\tan x+\cot x=\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}=\frac{\sin x}{-\frac{3}{2}\sin x}+\frac{-\frac{3}{2}\sin x}{\sin x}=-\frac{2}{3}-\frac{3}{2}=\frac{-4-9}{6}$ ۶

$=\boxed{-\frac{13}{6}}$

$1+\sin 50=\sin 90+\sin 50=2\sin\left(\frac{90+50}{2}\right)\times\cos\left(\frac{90-50}{2}\right)=2\underbrace{\sin 70}_{\cos 20}\times\cos 20$

$\sin p+\sin q=2\sin\left(\frac{p+q}{2}\right)\times\cos\left(\frac{p-q}{2}\right)$ $\quad =2\cos^2 20$

$\sin 50+\sin 10=2\sin\left(\frac{50+10}{2}\right)\times\cos\left(\frac{50-10}{2}\right)=2\underbrace{\sin 30}_{\frac{1}{2}}\times\cos 20^\circ=\cos 20$ ۷

$\frac{\sqrt{2\cos^2 20}}{\cos 20}=\frac{\sqrt{2}\,|\cos 20|}{\cos 20}\xrightarrow{\cos 20>0}\frac{\sqrt{2}\cos 20}{\cos 20}=\boxed{\sqrt{2}}$

$1-2\sin^2 5=1-\sin^2 5-\sin^2 5=\cos^2 5-\sin^2 5=\cos 10$

$2\cos^2 10-1=\cos^2 10-1+\cos^2 10=-\sin^2 10+\cos^2 10=\cos 20$

$\frac{1-\tan^2 20}{1+\tan^2 20}=\cos 40$

$\cos 10\cos 20\cos 40\times\frac{\sin 10}{\sin 10}=\frac{\overbrace{\overbrace{\overbrace{\sin 10\cos 10}^{\frac{1}{2}\sin 20}\times\cos 20}^{\frac{1}{4}\sin 40}\times\cos 40}^{\frac{1}{8}\sin 80}}{\sin 10}=\frac{1}{8}\,\frac{\overbrace{\sin 80}^{\cos 10}}{\sin 10}$ ۸

$=\boxed{\frac{1}{8}\cot 10^\circ}$

$\sin\alpha+3\cos\alpha=2\to\sin\alpha=2-3\cos\alpha$, $\sin^2\alpha+\cos^2\alpha=1$

$\to\cos^2\alpha+(2-3\cos\alpha)^2=1\to\cos^2\alpha+4+9\cos^2\alpha-12\cos\alpha=1$

$\to 10\cos^2\alpha-12\cos\alpha=-3$ (1)

$\cos^2\alpha=\frac{1+\cos 2\alpha}{2}\to\cos 2\alpha=2\cos^2\alpha-1\xrightarrow{\times 5}5\cos 2\alpha=10\cos^2\alpha-5$ ۹

$10\cos^2\alpha-12\cos\alpha-5=-3-5=\boxed{-8}$

طبق (1) → ۳- می‌شود

$\sin a\times\sin b=\frac{1}{2}(\cos(a-b)-\cos(a+b))\to\sin B\sin C=\frac{1}{2}(\cos(B-C)-\cos(B+C))$ (1)

$\cos a\times\cos b=\frac{1}{2}(\cos(a+b)+\cos(a-b))\to\cos\frac{A}{2}\times\cos\frac{A}{2}=\frac{1}{2}(\cos A+\cos 0)$

$A+B+C=180\to B+C=180-A\xrightarrow{(1)}\frac{1}{2}(\cos(180-(2C+A))-\cos(180-A))=\frac{1}{2}(\cos(2C+A)+\cos A)$

$\to B=180-C-A$

$\frac{1}{2}\cos A+\frac{1}{2}=-\frac{1}{2}\cos(2C+A)+\frac{1}{2}\cos A\to-\frac{1}{2}\cos(2C+A)=\frac{1}{2}\to\cos(2C+A)=-1$ ۱۰

$\to 2C+A=180\to C=B=72\to A=180-(2\times 72)=\boxed{36^\circ}$