نام و نام خانوادگی محمدرضا آخوندی پاسخنامه تشریحی تکلیف شماره ۱۱ ... کلاس ....

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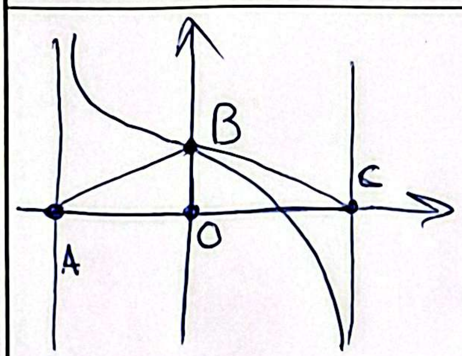

$A(+\frac{\pi}{8}, 0)$

$C(\frac{5\pi}{8}, 0)$

$B(0, 1)$

$S_{ABC} = \frac{AC \times BO}{2}$

$S_{ABC} = \frac{\frac{\pi}{2} \times 1}{2} = \boxed{\frac{\pi}{4}}$

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$\frac{4}{9} - \frac{2\sin\alpha}{3} - \frac{\cos^2\alpha}{4} = 0 \longrightarrow 9\cos^2\alpha + 24\sin\alpha = 16 \xrightarrow{\cos^2\alpha = 1 - \sin^2\alpha}$

$-9\sin^2\alpha + 24\sin\alpha - 7 = 0 \longrightarrow (24)^2 - (4)(-9)(-7) = 324 \longrightarrow \sqrt{324} = 18$

$\frac{-24 \pm 18}{-18} \rightarrow \boxed{\frac{1}{3}}$, $\frac{7}{3}$ ✗

$\boxed{\frac{1}{3}} \longrightarrow$ (triangle: hypotenuse 3, opposite side 1, adjacent side $2\sqrt{2}$, angle α) $\Big\}\ 1 + \tan^2\alpha = 1 + \left(\frac{1}{2\sqrt{2}}\right)^2 = \boxed{\frac{9}{8}}$

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$\frac{\sin\alpha}{-\cos\alpha} = 3 \rightarrow \sin\alpha = -3\cos\alpha \xrightarrow{\sin^2\alpha + \cos^2\alpha = 1} 10\cos^2\alpha = 1$

$\cos\alpha = \pm\frac{1}{\sqrt{10}} \longrightarrow \sin\alpha = \mp\frac{3}{\sqrt{10}}$ $\Big\}\ \sin\alpha - \cos\alpha = \begin{cases} -\frac{3}{\sqrt{10}} - \frac{1}{\sqrt{10}} = \boxed{-\frac{4}{\sqrt{10}}} \\ \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{10}} = \boxed{\frac{4}{\sqrt{10}}} \end{cases}$

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$\sin^4\alpha + \cos^2\alpha = \frac{4}{5} \longrightarrow \sin^4 - \sin^2 = -\frac{1}{5} \ /\ \sin^4 = \cos^4 - \cos2\alpha$

$\cos^4\alpha - \underbrace{\sin^2\alpha}_{1-\cos^2} = \cos2\alpha - \frac{1}{5} \longrightarrow \cos^4 + \cos^2 = \cos2\alpha + \frac{4}{5}$

$\xrightarrow{\cos2\alpha = \cos^2\alpha - \sin^2\alpha} \cos^4\alpha + \cos^2\alpha = \cos^2\alpha - \sin^2\alpha + \frac{4}{5} \rightarrow \boxed{\cos^4\alpha + \sin^2\alpha = \frac{4}{5}}$

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$3\sin x + \frac{\sin^2 x}{2} < 2\sin^2 x$

$6\sin x + \sin^2 x < 4\sin^2 x$

$\rightarrow \sin x < 0 \rightarrow x$ → ناحیه ۳ (circled) ، ناحیه ۴

$\frac{\sin^2 x - 1}{\cos x} > 0$

$\frac{-\cos^2 x}{\cos x} > 0 \rightarrow \cos x < 0$

$\rightarrow x$ → ناحیه ۳ (circled) ، ناحیه ۲

ناحیه سوم

$$\frac{2\cos x + 3\sin x}{\sin x \cos x} = 0 \longrightarrow 2\cos x = -3\sin x \Rightarrow \tan x = \frac{2}{-3}$$

$$\tan x + \cos x = \frac{1}{\frac{\sin x}{2}} = \frac{2}{\sin x} = \frac{2}{\frac{2\tan x}{1+\tan^2 x}} = \frac{2+2\tan^2 x}{2\tan x}$$

جواب: $-\frac{13}{6}$ ۶

$$\frac{\sqrt{1+\cos 40^\circ}}{\cos 40^\circ + \cos 80^\circ} = \frac{\sqrt{2\cos^2 \frac{80}{2}}}{2\cos\frac{120}{2}\cos\frac{40}{2}} = \frac{\sqrt{2\cos^2 \frac{\pi}{2}}}{\cos\frac{\pi}{2}} = \frac{\sqrt{2}\cos 40}{\cos 40} = \sqrt{2}$$

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$$(1-2\sin^2 a)(2\cos^2 10 - 1)\left(\frac{1-\tan^2 40}{1+\tan^2 40}\right) = (\cos 10)(\cos 40)(\cos 40)$$

$$(\cos 10)(\cos 20)(\cos 40) = \frac{\sin 80}{8\sin 10} = \frac{\cos 10}{8\sin 10} = \frac{1}{8}\cot 10^\circ = \frac{\cot 10^\circ}{8}$$

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$$5\sin x = 20 - 25\sin x \xrightarrow{\sin^2 x + \cos^2 x = 1} (20 - 25\cos x)^2 + \cos^2 x = 1$$

$$40\cos^2 x - 100\cos x + 25 = 0$$

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$$5\cos^2 x - 12\cos x = 5(2\cos^2 x - 1) - 12\cos x = 10\cos^2 x - 12\cos x - 5$$

$$10\cos^2 x - 12\cos x = -2 \longrightarrow -2 - 5 = -8$$

$$\left.\begin{aligned} \sin B \sin C &= \frac{\cos(B-C) + \cos A}{2} \\ \cos^2 \frac{A}{2} &= \frac{1+\cos A}{2} \end{aligned}\right\} \frac{\cos(B-C) + \cos A}{2} = \frac{1+\cos A}{2}$$

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$$A = 180 - (2 \times 72) = 36$$

$$\left.\begin{aligned} \cos(B-C) &= 1 \\ \cos 0^\circ &= 1 \end{aligned}\right\} B = C$$