

۱۔ جواب کی عبارات ملتان زیراً درست اور جو۔

(الف) $\cos 2x = 3 \sin x - 1$

(ب) $\cos 2x + \cos x - 2 = 0$

(الف) $\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 3\sin x - 1 \rightarrow 2\sin^2 x + 3\sin x - 2 = 0$

$\xrightarrow{\text{دو}} 2\sin^2 x + 3\sin x - 2 = 0 \rightarrow (\sin x + 2)(2\sin x - 1) = 0 \rightarrow \sin x = \frac{-3}{2} = -1.5 \text{ } \overline{\text{GOE}}$

$\sin x = \frac{1}{2} = \sin \frac{\pi}{6} \rightarrow \sin x = \frac{1}{2} \text{ } \overline{\text{GOE}}$

$x = 2k\pi + \frac{\pi}{6} \text{ } \overline{\text{GOE}}$

(ب) $\cos 2x + \cos x - 2 = 0 \rightarrow \cos^2 x - \sin^2 x + \cos x - 2 = 0 \rightarrow \cos^2 x + \cos x - 3 = 0$

$\xrightarrow{\text{دو}} \cos^2 x + \cos x - 3 = 0 \rightarrow (\cos x + 3)(\cos x - 1) = 0 \rightarrow \cos x = \frac{-3}{1} \text{ } \overline{\text{GOE}}$

$\cos x = 1 = \cos 0 \rightarrow x = 2k\pi \pm 0 = 2k\pi \text{ } \overline{\text{GOE}}$

۲۔ جواب کی عبارات ملتان زیراً درست اور جو۔

(الف) $\sin^2 x - \cos^2 x = \sin 4x$

(ب) $2 \cos^2 x + 2 \sin x \cos x = 1$

(الف) $\sin^2 x - \cos^2 x = \sin 4x \rightarrow -\cos 2x = 2 \sin 2x \cos 2x$

$\rightarrow (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) = 2 \sin 2x \cos 2x$

$\rightarrow -\frac{1}{2} = \sin 2x = \sin(-\frac{\pi}{6}) \rightarrow 2x = 2k\pi - \frac{\pi}{6} \rightarrow x = k\pi - \frac{\pi}{12} \text{ } \overline{\text{GOE}}$

$\rightarrow \cos 2x = 0 \rightarrow 2x = 2k\pi \pm \frac{\pi}{2} \rightarrow x = k\pi \text{ } \overline{\text{GOE}}$

(ب) $2 \cos^2 x + 2 \sin x \cos x = 1 \rightarrow \sin 2x = 1 - 2 \cos^2 x \rightarrow \sin 2x = -\cos 2x$

$\sin 2x = \cos(\pi - 2x) \rightarrow \sin 2x = \sin(\frac{\pi}{2} - (\pi - 2x)) \rightarrow \sin 2x = \sin(2x - \frac{\pi}{2})$

$\rightarrow 2x - \frac{\pi}{2} = 2k\pi + 2x \rightarrow \overline{\text{GOE}}$

$\rightarrow 2x - \frac{\pi}{2} = 2k\pi + \pi - 2x \rightarrow 4x = 2k\pi + \frac{3\pi}{2} \rightarrow x = \frac{k\pi}{2} + \frac{3\pi}{4} \text{ } \overline{\text{GOE}}$

۳- جواب کلی معادلات مثلثاتی زیر را بدست آورید.

الف) $\cot x (2 \cos x + 1) = \frac{1}{\sin x}$

ب) $2 \sin^2 x - \sin x \cos x + \cos^2 x = 2$

الف) $\cot x (2 \cos x + 1) = \frac{1}{\sin x} \rightarrow \frac{\cos x}{\sin x} (2 \cos x + 1) = \frac{1}{\sin x}$

$\rightarrow \sin x \neq 0$

$2 \cos^2 x + \cos x - 1 = 0 \xrightarrow{\text{جواب}} \cos^2 x + \cos x - 1 = 0 \rightarrow (\cos x + 2)(\cos x - 1) = 0$

$\rightarrow \begin{cases} \cos x = -\frac{1}{2} = -1 = \cos \pi \rightarrow x = 2k\pi + \pi \\ \cos x = \frac{1}{2} = \cos(\frac{\pi}{3}) \rightarrow x = 2k\pi \pm \frac{\pi}{3} \end{cases}$

ب) $2 \sin^2 x - \sin x \cos x + \cos^2 x = 2 \xrightarrow{\div \sin^2 x} 2 - \cot x + \cot^2 x = 2(1 + \cot^2 x)$

$\cot^2 x + \cot x = 0 \rightarrow \cot x (1 + \cot x) = 0$

$\begin{cases} \cot x = 0 = \cot \frac{\pi}{2} \rightarrow x = k\pi + \frac{\pi}{2} \\ \cot x = -1 = \cot(-\frac{\pi}{4}) \rightarrow x = k\pi - \frac{\pi}{4} \end{cases}$

۴- جواب کلی معادلات مثلثاتی زیر را بدست آورید.

الف) $\cos(\frac{\pi}{4} + x) \cos(x - \frac{\pi}{2}) = \frac{1}{4}$

ب) $\cos(2x - \frac{\pi}{4}) = -\sin 2x$

الف) $\cos(\frac{\pi}{4} + x) \cos(x - \frac{\pi}{2}) = \frac{1}{4} \rightarrow \sin(\frac{\pi}{4} - (\frac{\pi}{4} + x)) \cos(\frac{\pi}{4} - x) = \frac{1}{4}$

$\rightarrow \sin(\frac{\pi}{4} - x) \cos(\frac{\pi}{4} - x) = \frac{1}{4} \rightarrow \frac{1}{2} \sin(\frac{\pi}{2} - 2x) = \frac{1}{4} \rightarrow \sin(\frac{\pi}{2} - 2x) = \frac{1}{2} = \sin(\frac{\pi}{6})$

$\frac{\pi}{2} - 2x = 2k\pi + \frac{\pi}{6} \rightarrow x = -k\pi + \frac{\pi}{6}$

$\frac{\pi}{2} - 2x = 2k\pi + \pi - \frac{\pi}{6} \rightarrow x = -k\pi - \frac{\pi}{6}$

ب) $\cos(2x - \frac{\pi}{4}) = -\sin 2x \rightarrow \cos(2x - \frac{\pi}{4}) = \sin(-2x) \rightarrow \cos(2x - \frac{\pi}{4}) = \cos(\frac{\pi}{4} + 2x)$

$\rightarrow 2x - \frac{\pi}{4} = 2k\pi \pm (\frac{\pi}{4} + 2x)$

$\begin{cases} 2x - \frac{\pi}{4} = 2k\pi + \frac{\pi}{4} + 2x \rightarrow \frac{\pi}{4} = \frac{\pi}{4} + 2k\pi \\ 2x - \frac{\pi}{4} = 2k\pi - \frac{\pi}{4} + 2x \rightarrow 2x = 2k\pi - \frac{\sqrt{2}\pi}{\sqrt{2}} \end{cases}$

$x = k\pi - \frac{\sqrt{2}\pi}{\sqrt{2}}$

۵ - مجموع جواب معادله‌های زیر را بیابید.

$$\frac{\sin^2 x + \sin^2 x}{\sin^2 x} - \sin^2 x = \cos^2 x$$

$$\frac{\sin^2 x + \sin^2 x}{\sin^2 x} - \sin^2 x = \cos^2 x \rightarrow \frac{\sin^2 x}{\sin^2 x} + 1 = \sin^2 x + \cos^2 x$$

$$\rightarrow \frac{\sin^2 x}{\sin^2 x} = 0 \rightarrow \begin{cases} \sin^2 x = 0 = \sin(0) \rightarrow \begin{cases} x = 2k\pi + 0 \rightarrow x = \frac{k\pi}{1} \\ x = 2k\pi + \pi - 0 \rightarrow x = \frac{k\pi}{1} + \frac{\pi}{1} \end{cases} \\ \sin^2 x \neq 0 \neq \sin(0) \rightarrow \begin{cases} x \neq 2k\pi + 0 \rightarrow x \neq k\pi \\ x \neq 2k\pi + \pi - 0 \rightarrow x \neq k\pi + \frac{\pi}{1} \end{cases} \end{cases}$$

$$\Rightarrow \underline{x = \frac{k\pi}{1} + \frac{\pi}{1}}$$

۶ - قدر مطلق تفاضل جواب‌های معادله‌های زیر را بیابید: $[-\pi, \frac{\pi}{1}]$ (بسیار آسان)

$$\frac{\sin^2 x}{1 + \cos^2 x} = \frac{1}{\sin^2 x} + \cot^2 x$$

$$\frac{\sin^2 x}{1 + \cos^2 x} = \frac{1}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} \rightarrow \frac{\sin^2 x}{1 + \cos^2 x} = \frac{1 + \cos^2 x}{\sin^2 x} \rightarrow \sin^2 x = (1 + \cos^2 x)^2$$

$$\rightarrow 1 - \cos^2 x = \cos^2 x + 2\cos^2 x + 1 \rightarrow 2\cos^2 x + 2\cos^2 x = 0 \rightarrow 2\cos^2 x (1 + \cos^2 x) = 0$$

$$\rightarrow \begin{cases} \cos^2 x = 0 = \cos(\frac{\pi}{2}) \rightarrow x = 2k\pi \pm \frac{\pi}{2} \rightarrow x = 2k\pi \pm \pi \rightarrow x = 2k\pi + \pi \end{cases}$$

$$1 + \cos^2 x = 0 \rightarrow \text{خارج از دامنه است}$$

$$\sin^2 x \neq 0 \neq \sin(0) \rightarrow \begin{cases} x \neq 2k\pi + 0 \rightarrow x \neq 2k\pi \rightarrow x \neq 2k\pi \\ x \neq 2k\pi + \pi - 0 \rightarrow x \neq 2k\pi + \pi \rightarrow x \neq 2k\pi \end{cases}$$

$$\text{جوابها} \Rightarrow x = -\pi, \pi \rightarrow |-\pi - \pi| = 2\pi$$

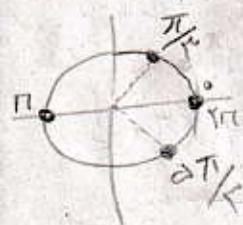
$$[-\pi, \frac{\pi}{1}] \quad \begin{aligned} K = -1 &\rightarrow x = -\pi \\ K = 0 &\rightarrow x = \pi \end{aligned}$$

۷ - تعداد جواب های معادله مثلثاتی زیر در فاصله $[0, 2\pi]$ را بیابید:

$$\cos^2 x - \sin^2 x \cos^2 x = 1$$

$$\cos^2 x - \sin^2 x \cos^2 x = 1 \rightarrow -\sin^2 x \cos^2 x = 1 - \cos^2 x$$

$$\rightarrow -\sin^2 x \cos^2 x = \sin^2 x \rightarrow \begin{cases} \cos^2 x = -1 = \cos \pi \rightarrow 2x = 2k\pi \pm \pi \rightarrow x = \frac{2k\pi}{2} \pm \frac{\pi}{2} \\ \sin^2 x = 0 \rightarrow \sin x = 0 = \sin(0) \rightarrow x = 2k\pi + 0 \\ x = 2k\pi + \pi \end{cases}$$



جواب ۵

$$k=0 \rightarrow \begin{matrix} x=0 \\ x=\pi \end{matrix} \quad \begin{matrix} x=\frac{\pi}{2} \\ x=\frac{3\pi}{2} \end{matrix}$$

$$k=1 \rightarrow \begin{matrix} x=2\pi \\ x=\frac{2\pi}{2} + \frac{\pi}{2} = \pi \end{matrix} \quad \begin{matrix} x=\frac{2\pi}{2} \\ x=\frac{2\pi}{2} - \frac{\pi}{2} = \frac{\pi}{2} \end{matrix}$$

$$k=2 \rightarrow \begin{matrix} x=\frac{4\pi}{2} + \frac{\pi}{2} = \frac{5\pi}{2} \\ x=\frac{4\pi}{2} - \frac{\pi}{2} = \frac{3\pi}{2} \end{matrix}$$

۸ - جواب کلی معادله مثلثاتی زیر را بیابید:

$$\frac{1 - \tan x}{1 + \tan x} = \tan 2x$$

$$\frac{1 - \tan x}{1 + \tan x} = \tan(\frac{\pi}{4} - x) = \tan 2x \rightarrow 3x = k\pi + \frac{\pi}{4} - x \rightarrow 4x = k\pi + \frac{\pi}{4}$$

$$x = \frac{k\pi}{4} + \frac{\pi}{16}$$

$$\cos x \neq 0 \rightarrow x \neq k\pi \pm \frac{\pi}{2}$$

$$\cos 2x \neq 0 \rightarrow 2x \neq k\pi \pm \frac{\pi}{2} \rightarrow x \neq \frac{k\pi}{2} \pm \frac{\pi}{4}$$

$$1 + \tan x \neq 0 \rightarrow \tan x \neq -1 = \tan(\frac{3\pi}{4}) \rightarrow x \neq k\pi + \frac{3\pi}{4}$$

۹- معادلات مثلثاتی در بازه $[0, \pi]$ چند جواب دارد

$$\sqrt{1 + \sin 2x} = \sqrt{2} \cos x$$

۳ جواب
 $x = \frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}$

$$\sqrt{\sin^2 x + \cos^2 x + 2 \sin x \cos x} = \sqrt{2} (\cos^2 x - \sin^2 x)$$

$$|\sin x + \cos x| = \sqrt{2} (\cos x - \sin x) (\cos x + \sin x)$$

$$x = \frac{3\pi}{4} \quad K=0$$

$$\sin x + \cos x = 0 \xrightarrow{\div \cos x} \tan x + 1 = 0 \rightarrow \tan x = -1 = \tan \frac{3\pi}{4} \rightarrow x = K\pi + \frac{3\pi}{4}$$

$$\sin x + \cos x > 0 \Rightarrow \cos x - \sin x = \frac{\sqrt{2}}{2} \rightarrow -\sqrt{2} \sin(x - \frac{\pi}{4}) = \frac{\sqrt{2}}{2} \rightarrow \sin(x - \frac{\pi}{4}) = -\frac{1}{2}$$

$$x - \frac{\pi}{4} = 2K\pi - \frac{\pi}{6} \rightarrow x = 2K\pi + \frac{\pi}{12} \quad / \quad x - \frac{\pi}{4} = 2K\pi + \pi + \frac{\pi}{6} \rightarrow x = 2K\pi + \frac{17\pi}{12}$$

$x = \frac{\pi}{12} \quad K=0$ خارج بازه $[0, \pi]$

$$\sin x + \cos x < 0 \Rightarrow \cos x - \sin x = -\frac{\sqrt{2}}{2} \rightarrow -\sqrt{2} \sin(x - \frac{\pi}{4}) = -\frac{\sqrt{2}}{2} \rightarrow \sin(x - \frac{\pi}{4}) = \frac{1}{2}$$

$$x - \frac{\pi}{4} = 2K\pi + \frac{\pi}{6} \rightarrow x = 2K\pi + \frac{5\pi}{12} \quad / \quad x - \frac{\pi}{4} = 2K\pi + \pi + \frac{\pi}{6} \rightarrow x = 2K\pi + \frac{13\pi}{12}$$

$K=0 \rightarrow x = \frac{5\pi}{12}$ خارج بازه $[0, \pi]$

۱۰- جواب های معادلات مثلثاتی زیر را در بازه $[0, \pi]$ بنویسید

الف) $\sin^4 x - \cos^4 x = 1$

ب) $\sin^3 x - \cos^3 x = -1$

الف) $\sin^4 x - \cos^4 x = 1 \rightarrow (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) = 1 \rightarrow -\cos 2x = 1$

$$\rightarrow \cos 2x = -1 = \cos \pi \rightarrow 2x = 2K\pi + \pi \rightarrow x = K\pi + \frac{\pi}{2}$$

در بازه $[0, \pi]$
 $\frac{\pi}{2}$ و $\frac{3\pi}{2}$

ب) $\sin^3 x - \cos^3 x = -1$

معادلات $\sin^3 x \pm \cos^3 x = \pm 1$ برای تعیین جواب های معادله نقاط مرزی را مورد بررسی قرار می دهیم

$x = 0, \frac{3\pi}{4}, \pi$
 در بازه $[0, \pi]$