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نام و نام خانوادگی ......[illegible]...... پاسخنامه تشریحی تکلیف شماره ..۱۷.. کلاس ....[illegible]....

۱	الف) $\cos 2x = 3\sin x - 1 \to 2\underbrace{\cos^2 x}_{(1-\sin^2 x)} - 1 = 3\sin x - 1 \to 2 - 2\sin^2 x - 1 = 3\sin x - 1$ $\to 2\sin^2 x + 3\sin x - 2 = 0 \to \sin x = \frac{-3\pm 5}{4}$ $\begin{cases}\sin x = \frac{1}{2} ✓ \\ \sin x = -2 \text{ غ ق ق}\end{cases}$ $\sin x = \sin\frac{\pi}{6} \to \begin{cases} x = 2k\pi + \frac{\pi}{6} \\ x = 2k\pi + \frac{5\pi}{6}\end{cases}$ ب) $\cos 2x + \cos x - 2 = 0 \to 2\cos^2 x - 1 + \cos x - 2 = 0 \to 2\cos^2 x + \cos x - 3 = 0$ $\to \begin{cases}\cos x = 1 ✓ \\ \cos x = -\frac{3}{2} \text{ غ ق ق}\end{cases}$ $\cos x = \cos 0 \to \boxed{x = 2k\pi}$	۲
۲	الف) $\sin^4 x - \cos^4 x = \sin 4x \to \underbrace{\sin^2 x - \cos^2 x}_{-\cos 2x} = 2\sin 2x \cos 2x \to 2\sin 2x\cos 2x + \cos 2x = 0$ $\to \cos 2x(2\sin 2x + 1) = 0 \to \cos 2x = 0 \to 2x = k\pi + \frac{\pi}{2} \to x = \frac{k\pi}{2} + \frac{\pi}{4}$ ب) $2\underbrace{\cos^2 x}_{1+\cos 2x} + \underbrace{2\sin x\cos x}_{\sin 2x} = 1 \to \sin 2x = -\frac{1}{2} \to \begin{cases}2x = 2k\pi - \frac{\pi}{6} \\ 2x = 2k\pi + \frac{7\pi}{6}\end{cases} \to \begin{cases} x = k\pi - \frac{\pi}{12} \\ x = k\pi + \frac{7\pi}{12}\end{cases}$ $\to \sin 2x = -\cos 2x \xrightarrow{\div \cos 2x} \tan 2x = -1$ $\to 2x = k\pi - \frac{\pi}{4} \to \boxed{x = \frac{k\pi}{2} - \frac{\pi}{8}}$	۲
۳	الف) $\cot x(2\cos x + 1) = \frac{1}{\sin x} \xrightarrow{\times \sin x} \cos x(2\cos x + 1) = 1 \to 2\cos^2 x + \cos x - 1 = 0$ $\to \begin{cases}\cos x = \frac{1}{2} \longrightarrow \boxed{x = 2k\pi \pm \frac{\pi}{3}} \\ \cos x = -1 \xrightarrow{\sin x \neq 0} \text{غ ق ق}\end{cases}$ $\cos x(\cos x + \sin x) = 0$ $\quad -\frac{1}{2}\ \frac{1}{2}(\cos 2x + \sin 2x) = 1$ $\begin{cases}\cos x = 0 \to x = k\pi + \frac{\pi}{2} \\ \cos x + \sin x = 0 \to x = k\pi - \frac{\pi}{4}\end{cases}$ ب) $2\sin^2 x - \sin x\cos x + \cos^2 x = 2 \to \sin^2 x - \sin x\cos x = 1 \to \frac{1}{2}(1 - \cos 2x) - \frac{1}{2}\sin 2x = 1$ $\to \cos 2x + \sin 2x = -1 \to \sqrt{2}\sin(2x + \frac{\pi}{4}) = -1 \to \sin(2x + \frac{\pi}{4}) = -\frac{\sqrt{2}}{2} \to \begin{cases}2x + \frac{\pi}{4} = 2k\pi + \frac{7\pi}{4} \\ 2x + \frac{\pi}{4} = 2k\pi + \frac{5\pi}{4}\end{cases}$ $\longrightarrow \boxed{x = k\pi + \frac{3\pi}{4}}, \boxed{x = k\pi + \frac{\pi}{2}}$	۱/۵
۴	الف) $\cos(\frac{\pi}{4} + x)\cos(x - \frac{\pi}{4}) = \frac{1}{4} \to \cos A\cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$ $\to \cos(\frac{\pi}{4} + x)\cos(x - \frac{\pi}{4}) = \frac{1}{2}[\cos(2x) + \cos(\frac{\pi}{2})] = \frac{1}{2}\cos 2x = \frac{1}{4} \to \cos 2x = \frac{1}{2}$ $\to 2x = 2k\pi \pm \frac{\pi}{3} \to x = k\pi \pm \frac{\pi}{6}$ ب) $\cos(2x - \frac{\pi}{9}) = -\sin 2x$ $\quad \sin x = \cos(\frac{\pi}{2} - x) \to -\sin 2x = \cos(\frac{\pi}{2} + 2x)$ $\to \cos(2x - \frac{\pi}{9}) = \cos(\frac{\pi}{2} + 2x) \to \begin{cases}2x - \frac{\pi}{9} = -\frac{\pi}{2} - 2x + 2k\pi \to x = \frac{-7\pi}{72} + \frac{k\pi}{2} \\ 2x - \frac{\pi}{9} = \frac{\pi}{2} + 2x + 2k\pi \ ✗\end{cases}$	۲
۵	$\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x \to 2\cos 2x + 1 - \underbrace{\sin^2 x}_{\cos^2 x} = \cos^2 x \to 2\cos 2x = 0$ $\frac{2\sin 2x\cos 2x + \sin 2x}{\sin 2x} = 2\cos 2x + 1$ $\qquad \to \cos 2x = 0$ $\to 2x = k\pi + \frac{\pi}{2}$ $\sin 2x \neq 0 \to 2x \neq k\pi \to x \neq \frac{k\pi}{2}$ $\qquad \to x = \frac{k\pi}{2} + \frac{\pi}{4},\ x \neq \frac{k\pi}{2}$	۲

$\sin^2\frac{x}{2} = (1+\cos\frac{x}{2})^2 \to \sin\frac{x}{2} = \pm(1+\cos\frac{x}{2}) \to \sin\frac{x}{2} = 1+\cos\frac{x}{2}$

$\to \sin\frac{x}{2}-\cos\frac{x}{2} = 1 \to \sin(\frac{x}{2}-\frac{\pi}{4}) = \frac{\sqrt{2}}{2} \to \frac{x}{2}-\frac{\pi}{4} = \frac{\pi}{4}+2k\pi \to x=\pi+4k\pi$ ✗

مخرج را صفر می‌کند

$\sin\frac{x}{2}+\cos\frac{x}{2} = -1 \to \sin(\frac{x}{2}+\frac{\pi}{4}) = -\frac{\sqrt{2}}{2} \to x = 4k\pi$ ✗ مخرج صفر می‌شود

$x = 4k\pi \pm \pi \xrightarrow{[-\pi,\frac{3\pi}{2}]} x=\pi,-\pi \to |\pi-(-\pi)| = 2\pi$

2

$\underbrace{\cos^2 x}_{1-\sin^2 x} - \sin^2 x\cos 3x = 1 \to \sin^2 x(1+\cos 3x) = 0 \to \sin^2 x = 0 \to \sin x = 0$

$\to x = k\pi \to x = 0,\pi,2\pi$

$1+\cos 3x = 0 \to \cos 3x = -1 \to 3x = 2k\pi+\pi \to x = \frac{2k\pi}{3}+\frac{\pi}{3} \to x = \frac{\pi}{3},\pi,\frac{5\pi}{3}$

2

$\xrightarrow{\text{اجتماع}} x = 0,\frac{\pi}{3},\pi,\frac{5\pi}{3},2\pi$

$\frac{1-\tan x}{1+\tan x} = \tan 3x \to \tan(\frac{\pi}{4}-x) = \tan 3x \to \frac{\pi}{4}-x = 3x+k\pi$

$\to 4x = \frac{\pi}{4}-k\pi \to x = \frac{\pi}{16}-\frac{k\pi}{4}$

1,5

$\tan(\frac{\pi}{4}-x) = \tan 3x \to \frac{\pi}{4}-x = k\pi+3x \to x = \frac{k\pi}{4}+\frac{\pi}{16}$

$1+\tan x \neq 0 \to \tan x \neq -1 \to x \neq -\frac{\pi}{4}+k\pi$

$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \to \sqrt{2}\cos 2x \geq 0 \to \cos 2x \geq 0 \quad , 1+\sin 2x \geq 0 \to \sin 2x \geq -1$

1

$\underbrace{1+\sin 2x = 2\cos^2 2x}_{2\sin^2 2x+\sin 2x-1=0} = 1+\cos 4x \to \sin 2x = \cos 2x \to \tan 2x = 1 \to 2x = k\pi+\frac{\pi}{4}$

$\sin 2x = -1 \to x = k\pi-\frac{\pi}{4} \to \frac{3\pi}{4}$

$x = \frac{k\pi}{2}+\frac{\pi}{8}$

$x = k\pi+\frac{\pi}{12} \to \frac{\pi}{12}$

الف) $\sin^2 x - \underbrace{\cos^2 x}_{1-\sin^2 x} = 1 \to 2\sin^2 x = 2 \to \sin^2 x = 1 \to \sin x = \pm 1 \to \begin{cases} \sin x = 1 \xrightarrow{[0,2\pi]} x = \frac{\pi}{2} \\ \sin x = -1 \to x = \frac{3\pi}{2} \end{cases}$

ب) $\sin^3 x - \cos^2 x = -1$

2

$x = 0$

$x = 2\pi$

$x = \frac{\pi}{2} \ , \ x = \frac{3\pi}{2}$