

$$\text{الف) } \cos 2x = 3 \sin x - 1 \rightarrow 2 \cos^2 x - 1 = 3 \sin x - 1 \rightarrow 2 - 2 \sin^2 x - 1 = 3 \sin x - 1$$

$$(1 - \sin^2 x)$$

$$\rightarrow 2 \sin^2 x + 3 \sin x - 2 = 0 \rightarrow \sin x = \frac{-3 \pm \Delta}{4} \quad \begin{cases} \sin x = \frac{1}{2} \checkmark & \sin x = \sin \frac{\pi}{6} \\ \sin x = -2 \text{ قیچی} & \rightarrow \begin{cases} x = 2k\pi + \frac{\pi}{6} \\ x = 2k\pi + \frac{5\pi}{6} \end{cases} \end{cases}$$

$$\text{ب) } \cos 2x + \cos x - 2 = 0 \rightarrow 2 \cos^2 x - 1 + \cos x - 2 = 0 \rightarrow 2 \cos^2 x + \cos x - 3 = 0$$

$$\rightarrow \begin{cases} \cos x = 1 \checkmark \\ \cos x = -\frac{3}{2} \text{ قیچی} \end{cases} \quad \cos x = \cos 0 \rightarrow x = 2k\pi$$

$$\text{الف) } \sin^2 x - \cos^2 x = \sin x \rightarrow \sin^2 x - \cos^2 x = 2 \sin x \cos x \rightarrow 2 \sin x \cos^2 x + \cos^2 x = 0$$

$$- \cos^2 x$$

$$\rightarrow \cos^2 x (2 \sin x + 1) = 0 \rightarrow \cos^2 x = 0 \rightarrow x = k\pi + \frac{\pi}{2} \rightarrow x = \frac{k\pi}{2} + \frac{\pi}{2}$$

$$\text{ب) } \frac{2 \cos^2 x + 2 \sin x \cos x}{1 + \cos^2 x} = 1 \quad \begin{cases} \rightarrow \sin x = \frac{1}{2} \rightarrow \begin{cases} x = 2k\pi - \frac{\pi}{6} \\ x = 2k\pi + \frac{\pi}{6} \end{cases} \\ \rightarrow \sin x = -\cos^2 x \xrightarrow{\div \cos^2 x} \tan x = -1 \rightarrow x = k\pi - \frac{\pi}{4} \rightarrow x = \frac{k\pi}{4} - \frac{\pi}{4} \end{cases}$$

$$\text{الف) } \cos x (1 + \cos x) = \frac{1}{\sin x} \xrightarrow{\sin x} \cos x (2 \cos^2 x + 1) = 1 \rightarrow 2 \cos^2 x + \cos x - 1 = 0$$

$$\rightarrow \begin{cases} \cos x = \frac{1}{2} \rightarrow x = 2k\pi \pm \frac{\pi}{3} \\ \cos x = -1 \xrightarrow{\sin x \neq 0} \text{ قیچی} \end{cases}$$

$$\frac{1}{2} - \frac{1}{2} (\cos^2 x + \sin^2 x) = 1$$

$$\text{ب) } 2 \sin^2 x - \sin x \cos x + \cos^2 x = 2 \rightarrow \sin^2 x - \sin x \cos x = 1 \rightarrow \frac{1}{2} (1 - \cos^2 x) - \frac{1}{2} \sin^2 x = 1$$

$$\rightarrow \cos^2 x + \sin x = -1 \rightarrow \sqrt{2} \sin(x + \frac{\pi}{4}) = -1 \rightarrow \sin(x + \frac{\pi}{4}) = -\frac{1}{\sqrt{2}} \rightarrow \begin{cases} x + \frac{\pi}{4} = 2k\pi + \frac{3\pi}{4} \\ x + \frac{\pi}{4} = 2k\pi + \frac{5\pi}{4} \end{cases}$$

$$\rightarrow \begin{cases} x = k\pi + \frac{\pi}{2} \\ x = k\pi + \frac{3\pi}{2} \end{cases}$$

$$\text{الف) } \cos(\frac{\pi}{4} + x) \cos(x - \frac{\pi}{4}) = \frac{1}{2} \rightarrow \cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$$

$$\rightarrow \cos(\frac{\pi}{4} + x) \cos(x - \frac{\pi}{4}) = \frac{1}{2} [\cos(\pi x) + \cos(\frac{\pi}{2})] = \frac{1}{2} \cos^2 x = \frac{1}{2} \rightarrow \cos^2 x = \frac{1}{2}$$

$$\rightarrow x = 2k\pi \pm \frac{\pi}{4} \rightarrow x = k\pi \pm \frac{\pi}{4}$$

$$\text{ب) } \cos(x - \frac{\pi}{4}) = -\sin x \quad \sin x = \cos(\frac{\pi}{4} - x) \rightarrow -\sin x = \cos(\frac{\pi}{4} + x)$$

$$\rightarrow \cos(x - \frac{\pi}{4}) = \cos(\frac{\pi}{4} + x) \rightarrow \begin{cases} x - \frac{\pi}{4} = -\frac{\pi}{4} - x + 2k\pi \rightarrow x = \frac{-\sqrt{2}}{\sqrt{2}} + \frac{k\pi}{2} \\ x - \frac{\pi}{4} = \frac{\pi}{4} + x + 2k\pi \end{cases}$$

$$\frac{\sin^2 x + \sin^2 x}{\sin^2 x} - \sin^2 x = \cos^2 x \rightarrow 2 \cos^2 x + 1 - \sin^2 x = \cos^2 x \rightarrow \cos^2 x = 0$$

$$\rightarrow \cos^2 x = 0$$

$$\frac{2 \sin^2 x \cos^2 x + \sin^2 x}{\sin^2 x} = 2 \cos^2 x + 1$$

$$\rightarrow x = k\pi + \frac{\pi}{2}$$

$$\sin^2 x \neq 0 \rightarrow x \neq k\pi \rightarrow x \neq \frac{k\pi}{2}$$

$$\rightarrow x = \frac{k\pi}{2} + \frac{\pi}{2}, x \neq \frac{k\pi}{2}$$

$$\sin \frac{x}{2} = (1 + \cos \frac{x}{2})^{\frac{1}{2}} \rightarrow \sin \frac{x}{2} = \pm (1 + \cos \frac{x}{2}) \rightarrow \sin \frac{x}{2} = 1 + \cos \frac{x}{2}$$

$$\rightarrow \sin \frac{x}{2} - \cos \frac{x}{2} = 1 \rightarrow \sin (\frac{x}{2} - \frac{\pi}{4}) = \frac{\sqrt{2}}{2} \rightarrow \frac{x}{2} - \frac{\pi}{4} = \frac{\pi}{4} + 2k\pi \rightarrow x = \pi + 4k\pi$$

$$\sin \frac{x}{2} + \cos \frac{x}{2} = -1 \rightarrow \sin (\frac{x}{2} + \frac{\pi}{4}) = -\frac{\sqrt{2}}{2} \rightarrow x = 4k\pi$$

$$x = 4k\pi \pm \pi \quad \left[-\pi, \frac{\pi}{2} \right] \quad x = \pi, -\pi \rightarrow |x - (-\pi)| = 2\pi$$

$$\frac{\cos^2 x - \sin^2 x}{1 - \sin^2 x} = 1 \rightarrow \sin^2 x (1 + \cos^2 x) = 0 \rightarrow \sin^2 x = 0 \rightarrow \sin x = 0$$

$$\rightarrow x = k\pi \rightarrow x = 0, \pi, 2\pi$$

$$1 + \cos^2 x = 0 \rightarrow \cos^2 x = -1 \rightarrow x = 2k\pi + \pi \rightarrow x = \frac{2k\pi}{2} + \frac{\pi}{2} \rightarrow x = \frac{\pi}{2}, \pi, \frac{3\pi}{2}$$

$$\xrightarrow{\text{نوع}} x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$$

$$\frac{1 - \tan x}{1 + \tan x} = \tan^2 x \rightarrow \tan (\frac{\pi}{4} - x) = \tan^2 x \rightarrow \frac{\pi}{4} - x = 2x + k\pi$$

$$\rightarrow 2x = \frac{\pi}{4} - k\pi \rightarrow x = \frac{\pi}{8} - \frac{k\pi}{2}$$

$$1 + \tan x \neq 0 \rightarrow \tan x \neq -1 \rightarrow x \neq -\frac{\pi}{4} + k\pi$$

$$\sqrt{1 + \sin^2 x} = \sqrt{2} \cos x \rightarrow \sqrt{2} \cos x \geq 0 \rightarrow \cos x \geq 0, 1 + \sin^2 x \geq 0 \rightarrow \sin^2 x \geq -1$$

$$1 + \sin^2 x = \sqrt{2} \cos x = 1 + \cos^2 x \rightarrow \sin^2 x = \cos^2 x \rightarrow \tan^2 x = 1 \rightarrow x = k\pi + \frac{\pi}{4}$$

$$x = \frac{k\pi}{2} + \frac{\pi}{4}$$

$$\omega) \frac{\sin^2 x - \cos^2 x}{1 - \sin^2 x} = 1 \rightarrow \sin^2 x = 1 \rightarrow \sin^2 x = 1 \rightarrow \sin x = \pm 1 \rightarrow \begin{cases} \sin x = 1 \rightarrow x = \frac{\pi}{2} \\ \sin x = -1 \rightarrow x = \frac{3\pi}{2} \end{cases}$$

$$\iota) \sin^2 x - \cos^2 x = -1$$

$$x = 0$$

$$x = \pi$$

$$x = \frac{\pi}{2}, x = \frac{3\pi}{2}$$