پارسا انوری — تکلیف ۱۶

۱- الف)

$\cos 2x = 3\sin x - 1 = 1 - 2\sin^2 x \Rightarrow 2\sin^2 x + 3\sin x - 2 = 0 = (2\sin x - 1)(\sin x + 2) \Rightarrow$

$\Rightarrow \sin x \begin{cases} -2 \ \text{✗} \\ \frac{1}{2} \end{cases} \Rightarrow x = \begin{cases} 2k\pi + \pi/6 \\ 2k\pi + 5\pi/6 \end{cases}$

ب)

$\cos 2x + \cos x - 2 = 2\cos^2 x + \cos x - 3 = (2\cos + 3)(\cos - 1) = 0 \Rightarrow$

$\cos x \begin{cases} -3/2 \ \text{✗} \\ 1 \end{cases} \Rightarrow x = 2k\pi$

۲- الف)

$\sin^4 x - \cos^4 x = (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) = -\cos 2x = \sin 4x = 2\sin 2x \cos 2x \Rightarrow$

$\sin 2x = -\frac{1}{2} \Rightarrow 2x = \begin{cases} 2k\pi + 7\pi/6 \\ 2k\pi + 11\pi/6 \end{cases} \Rightarrow x = \begin{cases} k\pi + \frac{7\pi}{12} \\ k\pi + \frac{11\pi}{12} \end{cases}$

ب)

$2\cos^2 x - 1 + 2\sin x \cos x = 0 = \cos 2x + \sin 2x = 0 \Rightarrow \cos 2x = \sin 2x \Rightarrow$

$\cos 2x = \cos(\pi/2 - 2x) \Rightarrow 2x \begin{cases} 2k\pi + \pi/2 - 2x \Rightarrow x = k\pi/2 + \pi/8 \\ 2k\pi - \pi/2 + 2x \ \text{✗} \end{cases}$

۳- الف)

$\frac{\cos x(2\cos x + 1)}{\sin x} = \frac{1}{\sin x} \xrightarrow{\sin x \neq 0} 2\cos^2 x + \cos x - 1 = 0 \longrightarrow$

$\Rightarrow (2\cos - 1)(\cos x + 1) = 0 \Rightarrow \cos x \begin{cases} 1/2 \ ✓ \\ -1 \ \text{✗} \end{cases}$ زیرا اگر $\cos x = -1 \Rightarrow \sin x = 0$ که خلاف شرط اول است.

$\cos x = 1/2 \Rightarrow x = 2k\pi \pm \pi/3$

ب)

$2\sin^2 x - \sin x \cos x + \cos^2 x = 2 \xrightarrow{\div \cos^2} 2\tan^2 x - \tan x + 1 = \frac{2}{\cos^2 x} = 2(1 + \tan^2 x) \Rightarrow$

$\Rightarrow 2\tan^2 - \tan + 1 = 2\tan^2 + 2 \Rightarrow \tan + 1 = 0 \Rightarrow \tan x = -1 \Rightarrow x = k\pi - \pi/4$

۴) $\cos(\pi/4 + x)\cos(x - \pi/4) \xrightarrow{\pi/4 + x - \pi/2 = x - \pi/4} \cos(\pi/4 + x)\cos(\pi/4 + x - \pi/2) \Rightarrow$

$\Rightarrow \cos(\pi/4 + x)\sin(\pi/4 + x) = \frac{1}{2}\sin(\pi/2 + 2x) = 1/4 \Rightarrow \cos 2x = 1/2$

الف) $\Rightarrow 2x = 2k\pi \pm \pi/3 \Rightarrow x = k\pi \pm \pi/6$

ب) $\cos(2x - \pi/9) = \sin(-2x) = \cos(\pi/2 + 2x) \Rightarrow 2x - \pi/9 = 2k\pi \pm (2x + \pi/2)$

$4x = 2k\pi - 7\pi/18 \Rightarrow x = k\pi/2 - \frac{7\pi}{72}$

۵- $\frac{2\sin 2x\cos 2x + \sin 2x}{\sin 2x} - \sin^2 = \frac{\sin 2x(2\cos 2x + 1)}{\sin 2x} - \sin^2 = 4\cos^2 x - 1 - \sin^2 x = \cos^2 x \Rightarrow$

$\xrightarrow{\sin^2 x = 1 - \cos^2 x} 4\cos^2 x - 2 = 0 \Rightarrow \cos x = \pm\frac{\sqrt{2}}{2} \Rightarrow x = k\pi \pm \pi/4 = k\pi/2 + \pi/4$

۶- نکته: $\frac{\sin x}{1 + \cos x} = \tan x/2$ (I) / $\frac{\sin x/2}{1 + \cos x/2} = \frac{1 + \cos x/2}{\sin x/2} \Rightarrow$

$\tan x/4 = \cot x/4 = \tan(\pi/2 - x/4) \Rightarrow x/4 = k\pi + \pi/2 - x/4 \Rightarrow x/2 = k\pi + \pi/2$

$\Rightarrow x = 2k\pi + \pi \Rightarrow x = -\pi, +\pi \Rightarrow |\pi - (-\pi)| = 2\pi$ ج

۷- $\cos^2 x - 1 = \sin^2 x\cos 2x = \sin x \xrightarrow{\sin x = 0} \cos 3x = 1 \Longrightarrow$

$3x = 2k\pi \Rightarrow x = 2k\pi/3$
$\sin x = 0 \Rightarrow x = k\pi$
$\Rightarrow x = 0, \pi, 2\pi/3, 2\pi$ ۴ جواب

۸- $\frac{1-tg_x}{1+tg_x} = \tan(\pi/4 - x) = \tan 3x \Rightarrow 3x = k\pi + \pi/4 - x \Rightarrow$

$\Rightarrow x = k\pi/4 + \pi/16$

۹- $1+\sin 2x = 2\cos^2 2x \Rightarrow \sin 2x = 2\cos^2 2x - 1 = \cos 4x \Rightarrow \cos 4x = \cos(\pi/2 - 2x)$

$\Rightarrow 4x = 2k\pi \pm (\pi/2 - 2x)$ $x \begin{cases} \frac{k\pi}{3} + \frac{\pi}{12} \\ k\pi - \pi/4 \end{cases} \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12}$

$\frac{3\pi}{4}$ یکی‌ان

۳ جواب دارد

۱۰- الف) $(\sin^2 + \cos^2)(\sin^2 - \cos^2) = 1 = -\cos 2x \Rightarrow \cos 2x = -1 \Rightarrow$

$\Rightarrow 2x = 2k\pi + \pi \Rightarrow x = k\pi + \pi/2 \Rightarrow$ $x = \pi/2 , 3\pi/2$

ب) حالت خاص باید و باید
مضارب x $\frac{\pi}{2}$ باید.
$\sin x = -1 \Rightarrow x = 2k\pi - \pi/2$
$\cos x = -1 \Rightarrow x = 2k\pi + \pi$ $\Rightarrow$

$x = \pi \quad \frac{3\pi}{2}$