حسین شیبانی

۱۲,۵

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الف) $\cos 2\alpha = 3\sin\alpha - 1 \longrightarrow \cos^2 - \sin^2\alpha = 3\sin\alpha - 1$

$1 - 2\sin^2\alpha = 3\sin\alpha - 1$

$2\sin^2\alpha + 3\sin\alpha - 2 = 0 \quad (\sin + 2)(\sin - \frac{1}{2}) = 0 \quad \sin\alpha = 2$ غ ق ق

$\sin\alpha = \frac{1}{2}$ ق ق $\rightarrow \boxed{\alpha = k\pi \pm \frac{\pi}{6}}$

۱,۵

$x = 2k\pi + \frac{\pi}{6}, \quad x = 2k\pi + \frac{5\pi}{6}$

ب) $\cos 2x + \cos x - 2 = 0 \quad \cos^2 x - \sin^2 x + \cos x - 2 = 0$

$2\cos^2 x + \cos x - 3 = 0 \quad (\cos x + \frac{3}{2})(\cos x - 1) = 0 \quad \cos x = -\frac{3}{2}$ غ ق ق

$\cos x = 1$ ق ق $\rightarrow x = 2k\pi$

۲-

الف) $\sin^4 x - \cos^4 x = \sin 4x \quad \sin^2 x \cos^2 x = \sin 4x \quad -\cos 2x = \sin 4x$

$-\cos 2x = 2\sin 2x \cos 2x \quad \cos 2x(2\sin 2x + 1) = 0 \quad \cos 2x = 0 \rightarrow 2x = 2k\pi \pm \frac{\pi}{2} \quad \boxed{x = \frac{k\pi}{2} \pm \frac{\pi}{4}}$

۱,۷۵

$\sin 2x = -\frac{1}{2} \quad 2x = 2k\pi + \pi + \frac{\pi}{6} \Rightarrow \boxed{x = k\pi + \frac{7\pi}{12}}$

$2x = 2k\pi - \frac{\pi}{6} \rightarrow \boxed{x = k\pi - \frac{\pi}{12}}$

ب) $2\cos^2 x + 2\sin x \cos x = 1$

$\cancel{2\cos^2 x - 1 + 2\sin x \cos x = 0}$

$\cos 2x + \sin 2x = 0$

$1 + \tan x = 0 \quad \tan x = -1 \longrightarrow 2x = k\pi - \frac{\pi}{4} \rightarrow \boxed{x = \frac{k\pi}{2} - \frac{\pi}{8}}$

۳-

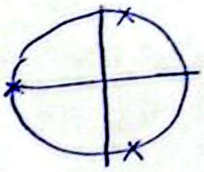
الف) $\cot x(2\cos x + 1) = \frac{1}{\sin x} \quad \frac{\cos x(2\cos x + 1)}{\cancel{\sin x}} = \frac{1}{\cancel{\sin x}} \quad 2\cos^2 x + \cos x - 1 = 0$

$(\cos x + 1)(\cos x - \frac{1}{2}) = 0$

۲

$\cos x = -1, \quad \cos x = \frac{1}{2}$

$\cos x = -1$ غ ق ق

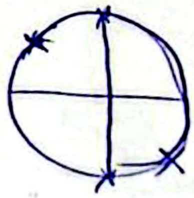


طبق دامنه $x \neq 2k\pi + \pi$ باشد

$\boxed{x = 2k\pi \pm \frac{\pi}{3}}$

ب) $2\sin^2 x - \sin x \cos x + \cos^2 x = 2 \quad \cos x(\cos x + \sin x) = 0 \quad \cos x = 0$

$\cos x = -\sin x$



$1 + \sin^2 x - \sin x \cos x = 2 \quad \cos^2 x + \sin x \cos x = 0$

$-\sin x \cos x = \frac{1 - \sin^2 x}{\cos^2 x}$

$\boxed{x = 2k\pi \pm \frac{\pi}{2}}$

$\boxed{x = k\pi - \frac{\pi}{4}}$

-4

1) $\cos\left(\frac{\pi}{4}+x\right)\cos\left(x-\frac{\pi}{4}\right)=\frac{1}{4}$ $\quad \sin\left(x-\frac{\pi}{4}\right)\cos\left(x-\frac{\pi}{4}\right)=\frac{1}{4}$ $\quad \frac{1}{2}\sin\left(2x-\frac{\pi}{2}\right)=\frac{1}{4}$

$\sin\left(2x-\frac{\pi}{2}\right)=-\sin\left(\frac{\pi}{2}-2x\right)=-\cos 2x=\frac{1}{2}$ $\quad \cos 2x=-\frac{1}{2}$

$2x=2k\pi+\pi\pm\frac{\pi}{3}$ $\quad \boxed{x=k\pi+\cancel{\frac{\pi}{2}}\pm\frac{\pi}{6}}$

1,5

2) $\cos\left(2x-\frac{\pi}{9}\right)=-\sin 2x$ $\quad 2x-\frac{\pi}{9}=\underset{2k}{2k\pi}+2x$ غ ق ق

$2x-\frac{\pi}{9}=2k\pi-2x$ $\quad 4x=2k\pi+\frac{\pi}{9}$ $\quad \boxed{\cancel{x=\frac{k\pi}{2}+\frac{\pi}{36}}}$

$x=\frac{k\pi}{2}-\frac{7\pi}{72}$

-5

$\frac{\sin 4x+\sin 2x}{\sin 2x}-\sin^2 x=\cos^2 x$ $\quad \frac{\sin 4x+\sin 2x}{\sin 2x}=1$

1

$2\sin 2x\cos 2x+\sin 2x=\sin 2x$ $\quad \cancel{\sin 2x(2\cos 2x+1)=\sin 2x}(2\cos 2x+1)=0$

$\cos 2x=-\cancel{\frac{1}{2}}\,0$ $\quad \cancel{2x=2k\pi+\pi\pm\frac{\pi}{3}}$ $\quad \boxed{\cancel{x=k\pi+\frac{\pi}{2}}\pm\frac{\pi}{6}}$ $\quad x=\frac{k\pi}{2}+\frac{\pi}{4}$

-6

$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}=\frac{1}{\sin\frac{x}{2}}+\cot\frac{x}{2}$ $\quad \frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}=\frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}$

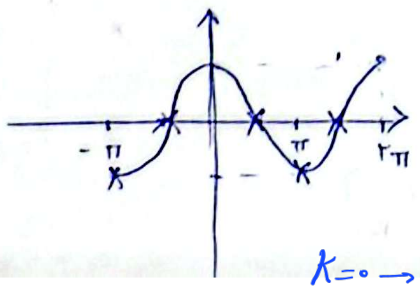
1

$\sin^2\frac{x}{2}=1+2\cos\frac{x}{2}+\cos^2\frac{x}{2}$

$\cos x+1+2\cos\frac{x}{2}=0$ $\quad 2\cos^2\frac{x}{2}+2\cos\frac{x}{2}=0$ $\quad 2\cos\frac{x}{2}\left(\cos\frac{x}{2}+1\right)=0$

$\cos\frac{x}{2}=-1$ $\quad \frac{x}{2}=2k\pi+\pi$ $\quad x=4k\pi+2\pi$

$\cos\frac{x}{2}=0$ $\quad \frac{x}{2}=2k\pi\pm\frac{\pi}{2}$ $\quad x=4k\pi\pm\pi$



$|-\pi|-|-\frac{\pi}{2}|-|\frac{\pi}{2}|-|\pi|-|\frac{3\pi}{2}|,\ \frac{3\pi}{2}$

$k=0\to x=\pi$ $\quad k=-1\to x=-\pi$ $\quad |-\pi-\pi|=2\pi$

۷-

$\cos^2 x - \sin^2 x \cos 2x = 1 \qquad -\sin^2 x \cos 2x = \sin^2 x \qquad [0, 2\pi]$

$-\cos 2x = 1 \qquad \cos 2x = -1 \qquad 2x = 2k\pi + \pi \qquad \boxed{x = \frac{2}{2}k\pi + \frac{\pi}{2}}$ ⇐ ۳ جواب داریم

۱,۵

$k=0 \to x = \frac{\pi}{3} \qquad k=2 \to x = \frac{5\pi}{3}$

$k=1 \to x = \pi \qquad 0,\ 2\pi$

$\{k \in W\}$

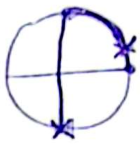
۸-

$\frac{1-\tan x}{1+\tan x} = \tan 3x$

۰

$\tan\left(\frac{\pi}{4} - x\right) = \tan 3x \to \frac{\pi}{4} - x = k\pi + 3x \to x = \frac{k\pi}{4} + \frac{\pi}{16}$

۹-

$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \quad , [0, \pi] \qquad \cos 2x > 0$  $\to \left[0, \frac{\pi}{4}\right]$

۱

$1 + \sin 2x = 2\cos^2 2x \to 1 + \sin 2x = \cancel{2(1 - \sin^2 2x)}$ ⇐ $2\sin^2 2x + \sin 2x - 1 = 0$

$2 - 2\sin^2 2x$

$(\sin 2x + \frac{2}{2})(\sin 2x - \frac{1}{2}) = 0 \qquad \sin 2x = -1 \qquad \sin 2x = \frac{1}{2} \qquad \boxed{x = \frac{\pi}{4} \ \text{غ ق ق}}$

$\sin 2x = -1 \to x = k\pi - \frac{\pi}{4} \to \frac{3\pi}{4}$

$x = k\pi + \frac{\pi}{12} \to \frac{\pi}{12}$

۱۰- در بازه $[0, 2\pi]$

الف) $\sin^4 x - \cos^4 x = 1$  $\boxed{x = 2k\pi \pm \frac{\pi}{2}}$ $\quad x = \frac{\pi}{2}, \frac{3\pi}{2}$

۱,۲۵

ب) $\sin^5 x - \cos^5 x = -1$  $\boxed{x = 2k\pi,\ 2k\pi - \frac{\pi}{2}}$ $\quad 2\pi, 0, \frac{3\pi}{2}$