حسین عیبازی

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الف) $\cos 2\alpha = 3\sin\alpha - 1 \longrightarrow \cos^2 - \sin^2\alpha = 3\sin\alpha - 1$

$1 - 2\sin^2\alpha = 3\sin\alpha - 1$

$2\sin^2\alpha + 3\sin\alpha - 2 = 0 \quad (\sin + 2)(\sin - \frac{1}{2}) = 0 \quad \sin\alpha = -2$ غ ق ق

$\sin\alpha = \frac{1}{2}$ ق ق $\rightarrow \boxed{\alpha = k\pi \pm \frac{\pi}{6}}$

ب) $\cos 2x + \cos x - 2 = 0 \quad \cos^2 x - \cancel{\sin^2 x} + \cos x - 2 = 0$

$2\cos^2 x + \cos x - 3 = 0 \quad (\cos x + \frac{3}{2})(\cos x - 1) = 0 \quad \cos x = -\frac{3}{2}$ غ ق ق

$\cos x = 1$ ق ق $\longrightarrow x = 2k\pi$

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الف) $\sin^4 x - \cos^4 x = \sin 4x \quad \sin^2 x - \cos^2 x = \sin 4x \quad -\cos 2x = -\sin 4x$

$-\cos 2x = -2\sin 2x\cos 2x \quad \cos 2x(2\sin 2x + 1) = 0 \quad \cos 2x = 0 \rightarrow 2x = 2k\pi \pm \frac{\pi}{2} \quad \boxed{x = k\pi \pm \frac{\pi}{4}}$

$\sin 2x = -\frac{1}{2} \quad 2x = 2k\pi + \pi + \frac{\pi}{6} = \boxed{x = k\pi + \frac{7\pi}{12}}$

$2x = 2k\pi - \frac{\pi}{6} \rightarrow \boxed{x = k\pi - \frac{\pi}{12}}$

ب) $2\cos^2 x + 2\sin x\cos x = 1 \quad \cancel{2\cos^2 x - 1 + 2\sin x\cos x} = 0$

$\cos 2x + \sin 2x = 0$

$1 + \tan x = 0 \quad \tan 2x = -1 \longrightarrow 2x = k\pi - \frac{\pi}{4} \rightarrow \boxed{x = \frac{k\pi}{2} - \frac{\pi}{8}}$

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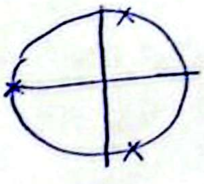
الف) $\cot x(2\cos x + 1) = \frac{1}{\sin x} \quad \frac{\cos x(2\cos x + 1)}{\cancel{\sin x}} = \frac{1}{\cancel{\sin x}} \quad 2\cos^2 x + \cos x - 1 = 0$

$(\cos x + \frac{2}{2})(\cos x - \frac{1}{2}) = 0$

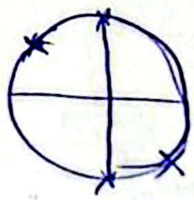
$\cos x = -1, \cos x = \frac{1}{2}$

$\cos x = -1$ غ ق ق



طبق دامنه مخرج $x \neq 2k\pi + \pi$ باشد

$\boxed{x = 2k\pi \pm \frac{\pi}{3}}$



ب) $2\sin^2 x - \sin x\cos x + \cos^2 x = 2 \quad \cos x(\cos x + \sin x) = 0 \quad \cos x = 0$

$\cos x = -\sin x$

$1 + \sin^2 x - \sin x\cos x = 2 \quad \cos^2 x + \sin x\cos x = 0$

$-\sin x\cos x = \underset{\cos^2 x}{\underline{1 - \sin^2 x}}$

$\boxed{x = 2k\pi \pm \frac{\pi}{2}}$

$\boxed{x = k\pi - \frac{\pi}{4}}$

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1) $\cos(\frac{\pi}{4}+x)\cos(x-\frac{\pi}{4}) = \frac{1}{4}$ $\quad \sin(x-\frac{\pi}{4})\cos(x-\frac{\pi}{4}) = \frac{1}{4}$ $\quad \frac{1}{2}\sin(2x-\frac{\pi}{2}) = \frac{1}{4}$

$\sin(2x-\frac{\pi}{2}) = -\sin(\frac{\pi}{2}-2x) = -\cos 2x = \frac{1}{2}$ $\quad \cos 2x = -\frac{1}{2}$

$2x = 2k\pi + \pi \pm \frac{\pi}{3}$ $\quad \boxed{x = k\pi + \frac{\pi}{2} \pm \frac{\pi}{6}}$

2) $\cos(2x-\frac{\pi}{9}) = -\sin 2x$ $\quad 2x - \frac{\pi}{9} = 2k\pi + 2x$ غ ق ق

$2x - \frac{\pi}{9} = 2k\pi - 2x$ $\quad 4x = 2k\pi + \frac{\pi}{9}$ $\quad \boxed{x = \frac{k\pi}{2} + \frac{\pi}{36}}$

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$\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x$ $\quad \frac{\sin 4x + \sin 2x}{\sin 2x} = 1$

$2\sin 2x\cos 2x + \sin 2x = \sin 2x$ $\quad \cancel{\sin 2x}(2\cos 2x + 1) = \cancel{\sin 2x}^{1}$ $\quad \sin 2x(2\cos 2x + 1) = 0$

$\cos 2x = -\frac{1}{2}$ $\quad 2x = 2k\pi + \pi \pm \frac{\pi}{3}$ $\quad \boxed{x = k\pi + \frac{\pi}{2} \pm \frac{\pi}{6}}$

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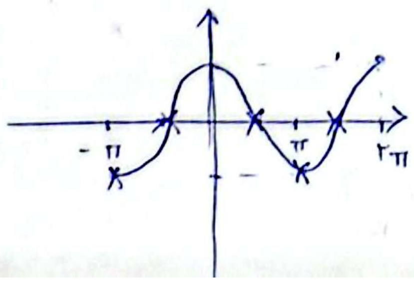
$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2}$ $\quad \frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}$

$\sin^2\frac{x}{2} = 1 + 2\cos\frac{x}{2} + \cos^2\frac{x}{2}$

$\cos x + 1 + 2\cos\frac{x}{2} = 0$ $\quad 2\cos^2\frac{x}{2} + 2\cos\frac{x}{2} = 0$ $\quad 2\cos\frac{x}{2}(\cos\frac{x}{2}+1) = 0$

$\cos\frac{x}{2} = -1$ $\quad \frac{x}{2} = 2k\pi + \pi$ $\quad x = 4k\pi + 2\pi$

$\cos\frac{x}{2} = 0$ $\quad \frac{x}{2} = 2k\pi \pm \frac{\pi}{2}$ $\quad x = 4k\pi \pm \pi$



$|-\pi| - |-\frac{\pi}{2}| - |\frac{\pi}{2}| - |\pi| - |\frac{3\pi}{2}| = \frac{3\pi}{2}$

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$\cos^2 x - \sin^2 x \cos 3x = 1 \qquad -\sin^2 x \cos 3x = \sin^2 x \qquad [0, 2\pi]$

$-\cos 3x = 1 \qquad \cos 3x = -1 \qquad 3x = 2k\pi + \pi \qquad \boxed{x = \frac{2}{3}k\pi + \frac{\pi}{3}}$ ⇐ ۳ جواب داریم

$k = 0 \rightarrow x = \frac{\pi}{3} \qquad k = 2 \rightarrow x = \frac{5\pi}{3}$

$k = 1 \rightarrow x = \pi$

$\{k \in W\}$

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$$\frac{1 - \tan x}{1 + \tan x} = \tan 3x$$

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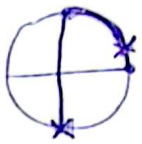
$\sqrt{1 + \sin 2x} = \sqrt{2} \cos 2x \quad , [0, \pi] \qquad \cos 2x > 0$



$\rightarrow [0, \frac{\pi}{2}]$

$1 + \sin 2x = 2\cos^2 2x \rightarrow 1 + \sin 2x = \cancel{2(1 - \sin^2 2x)} \Rightarrow 2\sin^2 2x + \sin 2x - 1 = 0$

$2 - 2\sin^2 2x$

$(\sin 2x + \frac{2}{2})(\sin 2x - \frac{1}{2}) = 0 \qquad \sin 2x = -1$

$\sin 2x = \frac{1}{2} \qquad \boxed{x = \frac{\pi}{4} \quad \text{ق ق}}$

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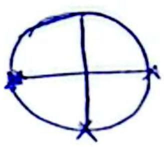
الف) $\sin^4 x - \cos^4 x = 1$

$\boxed{x = 2k\pi \pm \frac{\pi}{2}}$

ب) $\sin^2 x - \cos^2 x = -1$



$\boxed{x = 2k\pi , 2k\pi - \frac{\pi}{2}}$