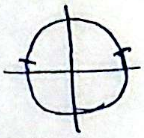
آرمین نوری

۱- الف) $\cos^2x - \sin^2x = 3\sin x - 1$ $\quad 1 - \sin^2x - \sin^2x = 3\sin x - 1$

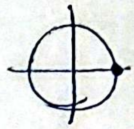
$2\sin^2x + 3\sin x - 2 = 0 \qquad \sin^2x + 3\sin x - 4 = 0 \rightarrow (\sin x + 4)(\sin x - 1) = 0$

$\sin x \begin{cases} \frac{-4}{2} = -2 \times \\ \frac{1}{2} = \frac{1}{2} \checkmark \end{cases} \qquad \sin x = \frac{1}{2}$



$\boxed{x = 2k\pi + \frac{\pi}{6} \,,\, 2k\pi + \frac{5\pi}{6}}$

ب) $\cos^2x - \sin^2x + \cos x - 2 = 0 \rightarrow 2\cos^2x + \cos x - 3 = 0 \qquad \cos x \begin{cases} 1 \checkmark \\ -\frac{3}{2} \times \end{cases}$



$\boxed{x = 2k\pi}$

۲- الف) $-(\cos^4x - \sin^4x) = 2\sin 2x \cos 2x \qquad -\cos 2x = 2\sin 2x \cos 2x$

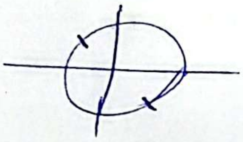
$2\sin 2x = -1 \qquad \sin 2x = -\frac{1}{2}$

$2x = 2k\pi - \frac{\pi}{6} \rightarrow x = k\pi - \frac{\pi}{12}$

$2x = 2k\pi - \frac{5\pi}{6} \rightarrow x = k\pi - \frac{5\pi}{12}$

ب) $2\cos^2x + 2\sin x\cos x = \sin^2x + \cos^2x \rightarrow \cos^2x - \sin^2x + 2\sin x\cos x = 0$

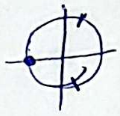
$\cos 2x + \sin 2x = 0$



$2x = k\pi - \frac{\pi}{4} \qquad x = \frac{k\pi}{2} - \frac{\pi}{8}$

۳- الف) $2\cos x + 1 = \frac{1}{\cos x} \rightarrow 2\cos^2x + \cos x = 1 \qquad 2\cos^2x + \cos x - 1 = 0$

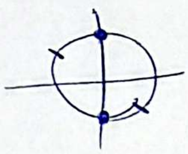
$\cos x \begin{cases} -1 \\ \frac{1}{2} \end{cases}$



$x = 2k\pi + \pi$ و $2k\pi \pm \frac{\pi}{3}$

ب) $2\sin^2x - \sin x\cos x + \cos^2x = 2\sin^2x + 2\cos^2x \rightarrow \cos^2x + \sin x\cos x = 0$

$\cos x(\cos x + \sin x) = 0 \rightarrow \begin{cases} \cos x = 0 \\ \cos x + \sin x = 0 \end{cases}$

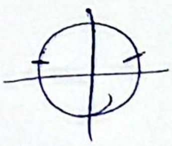


$x = k\pi - \frac{\pi}{4}$

$x = 2k\pi \pm \frac{\pi}{2}$

۴- الف) $\cos(\frac{\pi}{4} + x)\cos(\frac{\pi}{4} - x) = \frac{1}{4} \rightarrow \cos(\frac{\pi}{4} + x)\sin(\frac{\pi}{4} + x) = \frac{1}{4}$

$2\sin(\frac{\pi}{4} + x)\cos(\frac{\pi}{4} + x) = \frac{1}{2} \qquad \sin(2x + \frac{\pi}{2}) = \frac{1}{2}$



$2x + \frac{\pi}{2} = 2k\pi + \frac{\pi}{6}$

$2x + \frac{\pi}{2} = 2k\pi + \frac{5\pi}{6}$

$2x = 2k\pi - \frac{\pi}{3} \rightarrow x = k\pi - \frac{\pi}{6}$

$2x = 2k\pi + \frac{\pi}{3} \rightarrow x = k\pi + \frac{\pi}{6}$

$\boxed{x = k\pi \pm \frac{\pi}{6}}$

ب) $\cos(2x - \frac{\pi}{9}) = \sin(\frac{11\pi}{18} - 2x) \rightarrow \sin(\frac{11\pi}{18} - 2x) = -\sin 2x = \sin -2x$

$\sin(2x - \frac{11\pi}{18}) = \sin 2x$

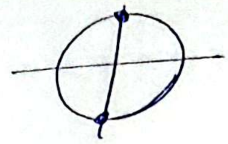
$2x = 2k\pi + \pi - 2x + \frac{11\pi}{18} \rightarrow 4x = 2k\pi + \frac{29\pi}{18}$

$\boxed{x = \frac{k\pi}{2} + \frac{29\pi}{72}}$

ارمین نوری

۵- $\frac{2\sin 2x\cos 2x+\sin 2x}{\sin 2x}-\sin^2 x=\cos^2 x \to \frac{\sin 2x(2\cos 2x+1)}{\sin 2x}=1$

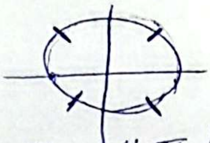
$2\cos 2x=0$ $\quad \cos 2x=0$



$2x=2k\pi\pm\frac{\pi}{2}$ $\quad x=k\pi\pm\pi/4$

۶- $\frac{2\sin x/4\cos x/4}{2\cos^2 x/4}=\frac{1+\cos x/2}{\sin x/2} \to \frac{\sin x/4}{\cos x/4}=\frac{2\cos^2 x/4}{2\sin x/4\cos x/4} \to \frac{\sin x/4}{\cos x/4}=\frac{\cos x/4}{\sin x/4}$

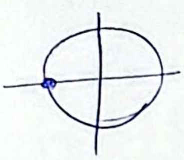
$\tan x/4=\cot x/4$



$x/4=\frac{k\pi}{2}+\pi/4$ $\quad x=2k\pi+\pi$

$x=-\pi$ و π $\quad \pi+\pi=2\pi$

۷- $\cos^2 x-\sin^2 x\cos 3x=\cos^2 x+\sin^2 x$ $\quad -\sin^2 x\cos 3x=\sin^2 x$ $\quad \cos 3x=-1$



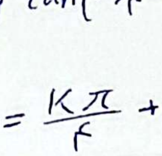
$3x=\pi$ $\quad x=\pi/3$

۸- $\frac{1-\tan x}{1+\tan x}=\frac{\tan\pi/4-\tan x}{1+\tan\pi/4\tan x}=\tan(\pi/4-x) \to \tan(\pi/4-x)=\tan 3x$

$3x=k\pi+\pi/4-x$ $\quad 4x=k\pi+\pi/4$ $\quad x=\frac{k\pi}{4}+\frac{\pi}{16}$

۹- $\sqrt{\sin^2 x+\cos^2 x+2\sin x\cos x}=\sqrt{2}\cos 2x$ $\quad \sqrt{(\sin x+\cos x)^2}=\sqrt{2}\cos 2x$

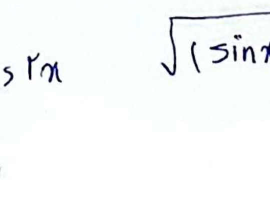
۱۰- الف) $\cos^4 x-\sin^4 x=-1$ $\quad \cos 2x=-1$



$2x=\pi$ $\quad x=\pi/2$

ب)

$\sin^3 x-\cos^3 x=-1$



$x=0, \frac{3\pi}{2}, 2\pi$