۱۴ شنبه

۱)

الف) $\cos 2x = 3\sin x - 1 \Rightarrow 1-2\sin^2 x = 3\sin x - 1$ ($\cos 2x = 1-2\sin^2 x$)

$\Rightarrow 2\sin^2 x + 3\sin x - 2 = 0 \Rightarrow \sin x = \frac{-3 \pm \sqrt{(3)^2 - 4(2)(-2)}}{2(2)} = \frac{-3 \pm 5}{4}$ → $\frac{1}{2} \Rightarrow \sin x = \frac{1}{2}$ ✓ ، $-2 \Rightarrow \sin x = -2$ غ ق ق

$\Rightarrow \boxed{x = 2k\pi + \frac{\pi}{6} \text{ یا } x = 2k\pi + \frac{5\pi}{6}}$

ب) $\underbrace{\cos 2x}_{2\cos^2 x - 1} + \cos x - 2 = 0 \Rightarrow 2\cos^2 x + \cos x - 3 = 0 \Rightarrow 2(\cos x + \frac{3}{2})(\cos x - 1) = 0$

$\Rightarrow \begin{cases} \cos x = 1 \Rightarrow \boxed{x = 2k\pi} \\ \text{یا} \\ \cos x = -\frac{3}{2} \text{ غ ق ق} \end{cases}$

۲)

الف) $\sin^4 x - \cos^4 x = \sin 4x = \underbrace{(\sin^2 x - \cos^2 x)}_{-\cos 2x}\underbrace{(\sin^2 x + \cos^2 x)}_{1} \Rightarrow -\cos 2x = \sin 4x$

$\Rightarrow -\cos 2x = 2\sin 2x \cos 2x \Rightarrow \begin{cases} \cos 2x = 0 \Rightarrow 2x = 2k\pi \pm \frac{\pi}{2} \Rightarrow \boxed{x = k\pi \pm \frac{\pi}{4}} \text{ و} \\ -1 = 2\sin 2x \Rightarrow \sin 2x = -\frac{1}{2} \Rightarrow \begin{cases} 2x = 2k\pi - \frac{\pi}{6} \Rightarrow \boxed{x = k\pi - \frac{\pi}{12}} \text{ و} \\ 2x = 2k\pi + \frac{7\pi}{6} \Rightarrow \boxed{x = k\pi + \frac{7\pi}{12}} \end{cases} \end{cases}$

ب) $\underbrace{2\sin x \cos x}_{\sin 2x} = \underbrace{1 - 2\cos^2 x}_{-\cos 2x} \Rightarrow \sin 2x = \cos 2x \Rightarrow \sin 2x = \sin(\frac{\pi}{2} + 2x)$

$\Rightarrow \begin{cases} 2x = 2k\pi + 2x + \frac{\pi}{2} \Rightarrow 0 = 2k\pi + \frac{\pi}{2} \text{ ✗} \\ 2x = 2k\pi + \pi \underbrace{- (2x + \frac{\pi}{2})}_{-2x - \frac{\pi}{2}} \Rightarrow 4x = 2k\pi + \frac{\pi}{2} \Rightarrow \boxed{x = \frac{k\pi}{2} + \frac{\pi}{8}} \end{cases}$

۳)

الف) $\Rightarrow \underbrace{\sin x \cot x}_{\cos x}(2\cos x + 1) = 1$ با توجه به اینکه سینوس در مخرج cot هست، cot x باید تعریف شده باشد

$\Rightarrow 1 = \cos x(2\cos x + 1) \Rightarrow 2\cos^2 x + \cos x - 1 = 0$ تنها شرط ما $\sin x \neq 0$ است!

$\Rightarrow 2(\cos x + 1)(\cos x - \frac{1}{2}) = 0 \Rightarrow \begin{cases} \cos x = -1 \Rightarrow \sin x = 0 \text{ در دامنه نیست!} \\ \cos x = \frac{1}{2} \Rightarrow \boxed{x = 2k\pi \pm \frac{\pi}{3}} \end{cases}$

ب) $2\sin^2 x - \underbrace{\sin x \cos x}_{\frac{1}{2}\sin 2x} + \underbrace{\cos^2 x}_{1 - \sin^2 x} = 2 \Rightarrow \sin^2 x - \frac{\sin 2x}{2} - 1 = 0 \Rightarrow 2\sin^2 x - 2 = \sin 2x$

$\Rightarrow -\underbrace{(1 - 2\sin^2 x)}_{\cos 2x} - 1 = \sin 2x \Rightarrow \sin 2x + \cos 2x = -1 \Rightarrow \begin{cases} 2x = 2k\pi + \pi \Rightarrow \boxed{x = k\pi + \frac{\pi}{2}} \\ 2x = 2k\pi - \frac{\pi}{2} \Rightarrow \boxed{x = k\pi - \frac{\pi}{4}} \end{cases}$

برای تناظر $2x = -\frac{\pi}{2}$

۴

الف) $\cos(\frac{\pi}{4}+x)\cos(x-\frac{\pi}{4})=\frac{1}{4} \Rightarrow \cos(x+\frac{\pi}{4})\sin(x+\frac{\pi}{4})=\frac{1}{4}$

$\left(\cos(x-\frac{\pi}{4})=\cos(\frac{\pi}{4}-x)=\cos(\frac{\pi}{2}-(\frac{\pi}{4}-x))=\sin(x+\frac{\pi}{4})\right)$ $\quad 2\cos(x+\frac{\pi}{4})\sin(x+\frac{\pi}{4})=\frac{1}{2}$

$\Rightarrow \sin(2(x+\frac{\pi}{4}))=\sin(2x+\frac{\pi}{2})=\frac{1}{2} \Rightarrow -\cos 2x=\frac{1}{2} \Rightarrow \cos 2x=-\frac{1}{2} \Rightarrow 2x=2k\pi\pm\frac{2\pi}{3} \Rightarrow \boxed{x=k\pi\pm\frac{\pi}{3}}$

ب) $\cos(2x-\frac{\pi}{9})=-\sin 2x=\sin(-2x)$

$\cos(2x-\frac{\pi}{9})=\cos(\frac{\pi}{9}-2x)=\sin(\frac{\pi}{2}-(\frac{\pi}{9}-2x))=\sin(\frac{7\pi}{18}+2x)$ $\Rightarrow \sin(\frac{7\pi}{18}+2x)=\sin(-2x)$

$(\cos\alpha=\cos(-\alpha))$ $\quad \cos(x)=\sin(\frac{\pi}{2}-x)$

$\Rightarrow \begin{cases} \frac{7\pi}{18}+2x=2k\pi-2x \Rightarrow 4x=2k\pi-\frac{7\pi}{18} \Rightarrow \boxed{x=\frac{k\pi}{2}-\frac{7\pi}{72}} \\ \frac{7\pi}{18}+2x=2k\pi+\pi+2x \Rightarrow \frac{7\pi}{18}=2k\pi+\pi \;\; \text{(غ ق ق)} \end{cases}$

۵

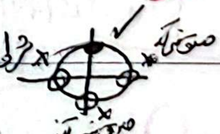
$\frac{\overset{2\sin 2x\cos 2x}{\sin 4x}+\sin 2x}{\sin 2x}=\underbrace{\cos^2 x+\sin^2 x}_{1} \Rightarrow \frac{\sin 2x(2\cos 2x+1)}{\sin 2x}=1 \Rightarrow 2\cos 2x+1=1 \Rightarrow 2\cos 2x=0$

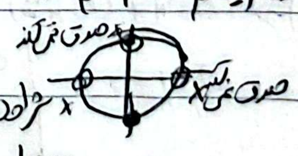
$\Rightarrow \cos 2x=0 \Rightarrow \sin 2x=\pm 1$ (حل جدول مثلثاتی) $\Rightarrow 2x=2k\pi\pm\frac{\pi}{2} \Rightarrow \boxed{x=k\pi\pm\frac{\pi}{4}}$

۶

$\frac{1}{\sin\frac{x}{2}}+\cot\frac{x}{2}=\frac{1}{\sin\frac{x}{2}}+\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}=\frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}$ با توجه به $\Rightarrow \frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}=\frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}$

$\Rightarrow (\sin\frac{x}{2})^2=(\cos\frac{x}{2}+1)^2 \Rightarrow \begin{cases} 1+\cos\frac{x}{2}=\sin\frac{x}{2} \Rightarrow \sin\frac{x}{2}-\cos\frac{x}{2}=1 \;\; \frac{x}{2}=2k\pi+\frac{\pi}{2} \Rightarrow \boxed{x=4k\pi+\pi} \\ 1+\cos\frac{x}{2}=-\sin\frac{x}{2} \Rightarrow \sin\frac{x}{2}+\cos\frac{x}{2}=-1 \Rightarrow x=2k\pi-\frac{\pi}{2} \Rightarrow \boxed{x=4k\pi-\pi} \end{cases}$





جواب اول: $x=4k\pi+\pi$

k	...	-1	0	1	...
x	...	-3π ✗	π ✓	5π ✗	...

جواب دوم: $x=4k\pi-\pi$

k	...	-1	0	1	...
x	...	-5π ✗	$-\pi$ ✓	3π ✗	...

$\Rightarrow$ جواب‌ها $= x=\pi, x=-\pi$

اختلاف جواب‌ها $=|\pi-(-\pi)|=\boxed{2\pi}$

$\cos^2 x - \sin^2 x \cos^3 x = 1 \Rightarrow$ ~~$\sin^2 x \cos^3 x = \dots$~~

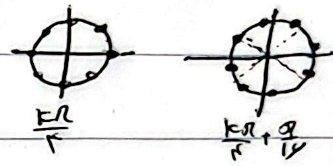
$\Rightarrow -\sin^2 x \cos^3 x = \underbrace{1-\cos^2 x}_{\sin^2 x} \Rightarrow$
① $\sin^2 x = 0 \Rightarrow \sin x = 0 \Rightarrow x = k\pi$
② $\cos^3 x = -1$
$\Rightarrow 3x = 2k\pi + \pi \Rightarrow x = \frac{2k\pi}{3} + \frac{\pi}{3}$

$k=0$	$k=1$ ✓
$x=0$	$x=\pi$

$k=1$ ✓	$k=2$ ✓
$x=\pi$	$x=\frac{5\pi}{3}$

$\tan(\frac{\pi}{4}-x) = \frac{\tan\frac{\pi}{4} - \tan x}{1+\tan\frac{\pi}{4}\tan x} = \frac{1-\tan x}{1+\tan x} \Rightarrow \tan(\frac{\pi}{4}-x) = \tan 3x$

$\Rightarrow 3x = k\pi + \frac{\pi}{4} - x \Rightarrow 4x = k\pi + \frac{\pi}{4} \Rightarrow x = \frac{k\pi}{4} + \frac{\pi}{16}$



همانطور که می‌بینید به ازای هر مقادیر داخلش تکرار شده

$\sqrt{1+\sin 2x} = \sqrt{2}\cos 3x \Rightarrow \sqrt{(\sin x+\cos x)^2} = \sqrt{2}\cos 3x \Rightarrow |\sin x+\cos x| = \sqrt{2}\cos 3x$

$\Rightarrow \sqrt{2}\,|\underbrace{\sin(x+\frac{\pi}{4})}_{\cos(\frac{\pi}{4}-x) = \cos(x-\frac{\pi}{4})}| = \sqrt{2}\cos 3x \Rightarrow |\cos(x-\frac{\pi}{4})| = \cos 3x \Rightarrow$
① $\cos(x-\frac{\pi}{4}) = \cos 3x$
② $\cos(x-\frac{\pi}{4}) = -\cos 3x$

$\Rightarrow$ ① $\Rightarrow \begin{cases} 3x = 2k\pi + x - \frac{\pi}{4} \Rightarrow \boxed{x = 2k\pi - \frac{\pi}{4}} \\ 3x = 2k\pi - x + \frac{\pi}{4} \Rightarrow \boxed{x = \frac{2k\pi}{4} + \frac{\pi}{16}} \end{cases}$

② $\Rightarrow \cos(x-\frac{\pi}{4}) = -\cos 3x = \cos(\pi - 3x) \Rightarrow \begin{cases} x - \frac{\pi}{4} = 2k\pi + \pi - 3x \Rightarrow 4x = 2k\pi + \frac{5\pi}{4} \Rightarrow \boxed{x = \frac{2k\pi}{4} + \frac{5\pi}{16}} \\ x - \frac{\pi}{4} = 2k\pi + 3x - \pi \Rightarrow -2x = 2k\pi - \frac{3\pi}{4} \Rightarrow \boxed{x = k\pi + \frac{3\pi}{8}} \end{cases}$

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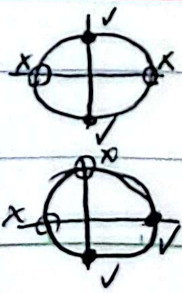
④ نقاط $\frac{k\pi}{2}$ را بررسی می‌کنیم:

الف) $\sin^4 x - \cos^4 x = 1$

ب) $\sin^3 x - \cos^3 x = 1$



$\Rightarrow x = k\pi + \frac{\pi}{2} \xrightarrow{\text{در دایره}} \boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$

$\Rightarrow \begin{cases} x = 2k\pi \\ x = 2k\pi + \frac{\pi}{2} \end{cases} \xrightarrow{\text{در بازه}} \boxed{x = 0, \frac{3\pi}{2}, 2\pi}$

PINE NOTEBOOK