نام خانوادگی ...... آرین پنجه فولادگران ...... پاسخنامه تشریحی تکلیف شماره ..۱۶.. کلاس ...... دوازدهم ریاضی A

الف) $\cos 2\alpha = 3\sin\alpha - 1 \rightarrow 1 - 2\sin^2\alpha = 3\sin\alpha - 1 \rightarrow 2\sin^2\alpha + 3\sin\alpha - 2 = 0$

$\rightarrow \sin^2\alpha + 3\sin\alpha - 4 = 0 \rightarrow (\sin\alpha + 2)(\sin\alpha - 1) = 0 \rightarrow \sin\alpha = \frac{1}{2}$ ✓ $\rightarrow \sin x = \sin\frac{\pi}{6}$

$\rightarrow x = 2k\pi + \frac{\pi}{6}$, $x = 2k\pi + \pi - \frac{\pi}{6} = \frac{5\pi}{6}$; $\sin\alpha = -\frac{4}{2} = -2$ ✗

ب) $\cos 2x + \cos x - 2 = 0 \rightarrow 2\cos^2 x + \cos x - 3 = 0 \rightarrow \cos^2 x + \cos x - 6 = 0$

$(\cos x + 3)(\cos x - 2) = 0 \rightarrow \cos x = \frac{2}{2} = 1 \rightarrow x = 2k\pi$; $\cos x = -\frac{3}{2}$ ✗

الف) $\sin^4\alpha - \cos^4\alpha = \sin 4x \rightarrow -\cos 2x = \sin 4x \rightarrow \sin 4x = \sin(\frac{3\pi}{2} + 2x)$

$4x = 2k\pi + \frac{3\pi}{2} + 2x \rightarrow x = k\pi + \frac{3\pi}{4}$, $4x = 2k\pi + \pi - \frac{3\pi}{2} - 2x \rightarrow x = \frac{k\pi}{3} - \frac{\pi}{12}$

ب) $2\cos^2 x + 2\sin x\cos x = 1 \rightarrow \cos 2x = -\sin 2x \rightarrow 2x = k\pi - \frac{\pi}{4}$

$\rightarrow x = \frac{k\pi}{2} - \frac{\pi}{8}$

الف) $\cot x(2\cos x + 1) = \frac{1}{\sin x} \Rightarrow \frac{\cos x}{\sin x}(2\cos x + 1) = \frac{1}{\sin x}$; $\sin x \neq 0 \rightarrow x \neq k\pi$

$2\cos^2 x + \cos x = 1 \rightarrow 2\cos^2 x - 1 = -\cos x \rightarrow \cos 2x = \cos(\pi - x) \rightarrow 3x = 2k\pi + \pi$

$\rightarrow x = \frac{2k\pi}{3} + \frac{\pi}{3}$, $x = 2k\pi - \pi \rightarrow x = 2k\pi - \pi$

ب) $2\sin^2 x - \sin x\cos x + \cos^2 x = 2 \xrightarrow{\div \sin^2 x} 2 - \cot x + \cot^2 x = 2(1 + \cot^2 x)$

$\cot^2 x + \cot x = 0 \rightarrow \cot x(\cot x + 1) = 0 \rightarrow \cot x = 0 \rightarrow x = k\pi + \frac{\pi}{2}$

$\cot x = -1 \rightarrow \cot x = \cot(-\frac{\pi}{4}) \rightarrow x = k\pi - \frac{\pi}{4}$; $x \neq k\pi$

الف) $\cos(\frac{\pi}{4} + x)\cos(\frac{\pi}{4} - x)$; $\frac{\pi}{4} - x + x + \frac{\pi}{4} = \frac{\pi}{2}$

۴) الف) $\cos(\frac{\pi}{4} + x)\sin(x + \frac{\pi}{4}) \rightarrow \frac{1}{2}\sin(2x + \frac{\pi}{2}) = \frac{1}{4} \rightarrow \cos 2x = \frac{1}{2} = \cos\frac{\pi}{3}$

$2x = 2k\pi \pm \frac{\pi}{3} \rightarrow x = k\pi + \frac{\pi}{6}$, $x = k\pi - \frac{\pi}{6}$

ب) $\cos(2x - \frac{\pi}{9}) = -\sin 3x \rightarrow \cos(2x - \frac{\pi}{9}) = \cos(\frac{\pi}{2} + 3x)$

$2x - \frac{\pi}{9} = 2k\pi + \frac{\pi}{2} + 3x$ ✗

$2x - \frac{\pi}{9} = 2k\pi - 3x - \frac{\pi}{2} + \frac{\pi}{9} \rightarrow x = \frac{k\pi}{2} - \frac{\pi}{8} + \frac{\pi}{36}$

$\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x \rightarrow \frac{2\sin 2x\cos 2x + \sin 2x}{\sin 2x} - \sin^2 x$; $\sin 2x \neq 0$

$2\cos 2x + 1 - \sin^2 x = \cos^2 x + \sin^2 x \rightarrow 2\cos 2x = 1 \rightarrow \cos 2x = \frac{1}{2} = \cos\frac{\pi}{3}$

$2x = 2k\pi + \frac{\pi}{3} \rightarrow x = k\pi + \frac{\pi}{6}$

$2x = 2k\pi - \frac{\pi}{3} \rightarrow x = k\pi - \frac{\pi}{6}$

۶)

$$\frac{\sin\frac{\alpha}{2}}{1+\cos\frac{\alpha}{2}} = \frac{1}{\sin\frac{\alpha}{2}} - \frac{\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}} \longrightarrow \tan\frac{\alpha}{4} = \frac{\cos\frac{\alpha}{2}+1}{\sin\frac{\alpha}{2}} \longrightarrow \frac{\sin\frac{\alpha}{4}}{\cos\frac{\alpha}{4}} = \frac{\cos\frac{\alpha}{2}+1}{\sin\frac{\alpha}{2}}$$

$$\sin\frac{\alpha}{4}\sin\frac{\alpha}{2} - \cos\frac{\alpha}{4}\cos\frac{\alpha}{2} = \cos\frac{\alpha}{4} \longrightarrow \cos\left(\frac{3\alpha}{4}\right) = \cos\frac{\alpha}{4} \longrightarrow$$

$$\cos\left(\frac{\alpha}{2}+\frac{\alpha}{2}\right)$$

$$\cos 3\frac{\alpha}{4} = \cos\left(\alpha - \frac{\alpha}{4}\right) \longrightarrow \frac{3\alpha}{4} = 2k\alpha + \alpha - \frac{\alpha}{4}$$

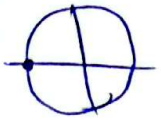
$$x = 2k\alpha + \alpha \longrightarrow \alpha = \alpha - \alpha$$

$$\alpha = 4k\alpha + 2\alpha \longrightarrow \quad |\alpha - (-\alpha)| = 2\alpha$$

۷)

$$\cos^4\alpha - \sin^2\alpha\cos 2\alpha = 1 \longrightarrow \sin^4\alpha = -\sin^2\alpha\cos 2\alpha \quad \sin\alpha = 0 \quad \cos 2\alpha = -1$$

$$1 - \sin^2\alpha - \sin^2\alpha\cos 2\alpha = 1$$



$$\longrightarrow \quad 3\alpha = 2k\alpha + \alpha \longrightarrow \alpha = \frac{2k\alpha}{3} + \frac{\alpha}{3}$$

$$\frac{\alpha}{3}, \quad \frac{\alpha}{3} + \frac{2\pi}{3}$$

۳ جواب در بازه $[0, 2\alpha)$ دارد.

۸)

$$\frac{1-\tan\alpha}{1+\tan\alpha} = \tan\left(\frac{\alpha}{4} - \alpha\right) = \tan 3\alpha \longrightarrow 3\alpha = k\alpha + \frac{\alpha}{4} - \alpha \longrightarrow \alpha = \frac{k\alpha}{4} + \frac{\alpha}{16}$$

۹)

$$\sqrt{1+\sin 2\alpha} = \sqrt{2\cos 2\alpha} \xrightarrow{\text{به توان ۲}} 1+\sin 2\alpha = 2\cos^2 2\alpha - 1 \quad \cos 2\alpha$$

$$\sin 2\alpha = \cos 2\alpha \longrightarrow \cos\left(\frac{\alpha}{2} - 2\alpha\right) = \cos 2\alpha \longrightarrow 2\alpha = 2k\alpha + \frac{\alpha}{2} - 2\alpha \longrightarrow \alpha = \frac{4k\alpha}{12} + \frac{\alpha}{12}$$

$$2\alpha = 2k\alpha + 2\alpha - \frac{\alpha}{2} \longrightarrow \quad k = 2k\alpha - \frac{\alpha}{4} \longrightarrow$$

$$1+\sin 2\alpha > 0$$

$$\cos 2\alpha > 0$$

$$k = 2k+1 \longrightarrow \frac{3\alpha}{4}$$

$\frac{\alpha}{12}$ ✓

$\frac{9\alpha}{12}$ ✗ ← ریشه نیست

$\frac{17\alpha}{12}$ ✓

۳ جواب در بازه $[0, 2\alpha)$ دارد.

۱۰)

الف) $\sin^4\alpha - \cos^4\alpha = 1 \longrightarrow -\cos 2\alpha \longrightarrow \cos 2\alpha = -1$

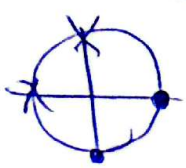
$$2\alpha = 2k\alpha + \alpha \longrightarrow \alpha = k\alpha + \frac{\alpha}{2}$$

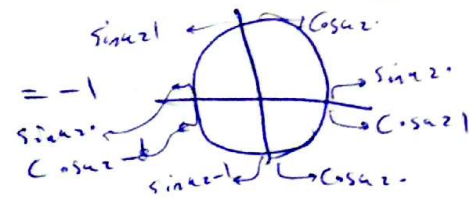
$\frac{\alpha}{2}$

$\frac{3\alpha}{2}$

$[0, 4\alpha)$

ب) $\sin^2\alpha - \cos^2\alpha = -1$





وجود ندارد.

$$\alpha = 2k\alpha$$

$0, 2\alpha$

$$\alpha = 2k\alpha - \frac{\alpha}{2}$$

$\frac{3\alpha}{2}$

← ۲ جواب