11,25

کامیار بهری حسابان دوازدهم B

۱)

ت) $\cos^2\alpha - \sin^2\alpha + \cos\alpha - 2 = 0$

$\cos^2\alpha - (1-\cos^2\alpha) + \cos x - 2 = 0$

$\cos^2\alpha - 1 + \cos^2\alpha + \cos\alpha - 2 = 0$

2

$2\cos^2\alpha + \cos\alpha - 3 = 0$

$\cos^2\alpha + \cos\alpha - 6 = 0$

$(\cos\alpha + 3)(\cos\alpha - 2) = 0$

$\cos\alpha = \frac{-3}{2}$ غ ق ق $\quad \cos\alpha = \frac{2}{2} = 1$

$\Rightarrow \cos\alpha = 1$

$\Rightarrow \boxed{x = 2k\pi}$ ✓

الف) $\cos^2\alpha - \sin^2\alpha = 3\sin\alpha - 1$

$\Rightarrow 1 - \sin^2\alpha - \sin^2\alpha = 3\sin x - 1$

$1 - 2\sin^2 x = 3\sin x - 1$

$2\sin^2\alpha + 3\sin\alpha - 2 = 0$

$\sin^2\alpha + 3\sin\alpha - 4 = 0$

$(\sin\alpha + 4)(\sin\alpha - 1) = 0$

$\Rightarrow \sin\alpha = \frac{-4}{2}$ غ ق ق $\quad \sin\alpha = \frac{1}{2}$

$\Rightarrow \sin\alpha = \frac{1}{2} \Rightarrow$ 150° 30°

$\Rightarrow \alpha = 2k\pi + \frac{\pi}{6}$ ①

$\alpha = 2k\pi + \pi - \frac{\pi}{6}$ ② ✓

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۲) 1,25

$(\sin^2 x - \cos^2 x)\underbrace{(\cancel{\sin^2 x + \cos^2 x})}_{①} = \sin 4x \qquad (\sin\alpha - \cos x)(\sin x + \cos x) = 4\sin 4x$

$\Rightarrow (\sin x - \cos x)(\sin x + \cos x) = 2\sin 2x\cos 2x = (4\sin x\cos x)(\cos^2 x - \sin^2 x)$

$\cancel{(\sin x - \cos x)}\underbrace{(\cancel{\sin x + \cos x})}_{①} = \underbrace{\cancel{(\cos x - \sin x)}}_{-1}\cancel{(\cos x + \sin x)}(4\sin x\cos x)$

$\Rightarrow -4\sin x\cos x = 1 \quad \cos 2x = 0 \to x = \frac{k\pi}{2} + \frac{\pi}{4}$

$\Rightarrow -2(2\sin x\cos x) = 1$

$\Rightarrow \sin 2x = \frac{-1}{2} \Rightarrow$ 210° 330°

$\Rightarrow 2x = 2k\pi - \frac{\pi}{6} \Rightarrow x = k\pi - \frac{\pi}{12}$ ✓

$2x = 2k\pi + \pi + \frac{\pi}{6} \Rightarrow x = k\pi + \frac{7\pi}{12}$ ✓

$\frac{2\cos^2\alpha}{\cos^2\alpha} + \frac{2\sin\alpha\cos\alpha}{\cos^2\alpha} = \frac{1}{\cos^2\alpha} \quad \div\cos^2\alpha \Rightarrow 2 + 2\tan\alpha = \frac{1}{\cos^2\alpha}$

$2(1+\tan\alpha) = 1 + \tan^2\alpha \qquad 2 = 1 - \tan\alpha$

$\Rightarrow 1 = -\tan\alpha \Rightarrow \tan\alpha = -1$

$\Rightarrow \alpha = k\pi - \frac{\pi}{4}$ ✓

$\cos 2x + \sin 2x = 0$

$1 + \tan 2x = 0 \to x = \frac{k\pi}{2} - \frac{\pi}{8}$

$$2\cos\theta+1=\frac{1}{\frac{\frac{\sin\theta}{1}}{\frac{\cos\theta}{\sin\theta}}}=\frac{1}{\cos\theta} \Rightarrow 2\cos\theta+1=\frac{1}{\cos\theta} \Rightarrow 2\cos^2\theta+\cos\theta=1$$

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$$2\cos^2\theta+\cos\theta-1=0 \Rightarrow \cos^2\theta+\cos\theta-2=0 \Rightarrow (\cos\theta+2)(\cos\theta-1)=0$$

الف)

$$\Rightarrow \left.\begin{array}{l}\cos\theta=\frac{-2}{2}=-1\\ \cos\theta=\frac{1}{2}=\frac{1}{2}\end{array}\right\} \Rightarrow \begin{array}{l}\theta=2k\pi+\pi \ \times \ \sin x\neq 0\\ \theta=2k\pi+\frac{\pi}{3} \ , \ 2k\pi-\frac{\pi}{3} \ \checkmark\end{array}$$

$$2\sin^2\alpha+\cos^2\alpha-\sin\alpha\cos\alpha=2 \xrightarrow{\div \sin^2\alpha} 2+\cot^2\alpha-\cot\alpha=2(1+\cot^2\alpha)$$

$$\cancel{2}+\cot^2\alpha-\cot\alpha=\cancel{2}+2\cot^2\alpha$$

ب)

$$\cancel{\cot\alpha}(\cot\alpha-1)=2\cancel{\cot^2\alpha} \Rightarrow \cot\alpha-1=2\cot\alpha$$
$$-1=\cot\alpha$$

$$\Rightarrow \left(\theta=k\pi-\frac{\pi}{4}\right) \checkmark$$

(۴ الف)

$$\left(\cos\left(\frac{\pi}{4}\right)\cos\theta-\sin\frac{\pi}{4}\sin\theta\right)\left(\cos\frac{\pi}{4}\cos\theta+\sin\frac{\pi}{4}\sin\theta\right)$$

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$$\Rightarrow=\left(\frac{\sqrt{2}}{2}\cos\theta\right)^2-\left(\frac{\sqrt{2}}{2}\sin\theta\right)^2 \Rightarrow \frac{\cos^2\theta-\sin^2\theta}{2}=\frac{1}{4}$$

$$\Rightarrow \cos 2\theta=\frac{1}{2} \Rightarrow 2\theta=2k\pi+\frac{\pi}{3} \Rightarrow \theta=k\pi\pm\frac{\pi}{6}$$
$$2\theta=2k\pi+\frac{2\pi}{2}-\frac{\pi}{3} \Rightarrow \theta=\cancel{k\pi+\frac{\pi}{3}} \ \checkmark$$

ب)

$$\cos\left(2x-\frac{\pi}{9}\right)=\cos\left(\frac{\pi}{2}+2x\right) \longrightarrow x=\frac{k\pi}{2}-\frac{7\pi}{72}$$

$$\frac{2\sin 2\alpha\cos 2\alpha}{\cancel{\sin 2\alpha}} + \frac{\sin 2\alpha}{\sin 2\alpha} - \sin^2\alpha = \cos^2\alpha$$

۵)

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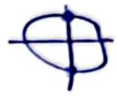
$$2\cos 2\alpha + 1 = \underbrace{\cancel{\cos^2\alpha + \sin^2\alpha}}_{1} \Rightarrow 2\cos 2\alpha = 0$$

$$\cos 2\alpha = 0$$



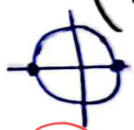
$$2\alpha = k\pi + \frac{\pi}{2} \Rightarrow \boxed{\alpha = \frac{k\pi}{2} + \frac{\pi}{4}}$$

$$\frac{a}{2} = \alpha \Rightarrow \frac{\sin\alpha}{1+\cos\alpha} = \frac{1+\cos\alpha}{\sin\alpha} \Rightarrow \sin^2\alpha = 1+\cos^2\alpha + 2\cos\alpha$$

$$\Rightarrow 1+\cos^2\alpha + 2\cos\alpha - 1 + \cos^2\alpha = 0$$

$$\Rightarrow 2\cos^2\alpha + 2\cos\alpha = 0 \Rightarrow 2\cos\alpha(\cos\alpha + 1) = 0$$

۶)



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$$\Rightarrow \cos\frac{a}{2} = -1 \Rightarrow \frac{a}{2} = 2k\pi + \pi \qquad a = 4k\pi + 2\pi$$

$$\cos\frac{a}{2} = 0 \Rightarrow \frac{a}{2} = k\pi + \frac{\pi}{2} \qquad a = 2k\pi + \pi$$

$$\Rightarrow |2\pi - \pi| = \pi$$

$k=0 \to x=\pi$ $\quad$ $k=-1 \to x=-\pi$ $\quad$ $|-\pi-\pi| = 2\pi$

۷)

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$$\cos^2\alpha - \sin^2\alpha\cos 3\alpha = 1 \Rightarrow -\sin^2\alpha\cos 3\alpha = \underbrace{\cancel{1-\cos^2\alpha}}_{\sin^2\alpha}$$

$$-\cancel{\sin^2\alpha}\cos 3\alpha = \cancel{\sin^2\alpha} \Rightarrow -\cos 3\alpha = 1$$

$$\Rightarrow \underline{\cos 3\alpha = -1}$$

$\Rightarrow$  $\Rightarrow 3\alpha = 2k\pi + \pi$

$$\boxed{\alpha = \frac{2}{3}k\pi + \frac{\pi}{3}}$$

 ⇐ ۲ جواب

جمع () کل عبارت به دست آمده است

$\sin x = 0 \to x = 0, \pi, 2\pi$

$\cos 3x = -1 \to x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

۸)

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$$\tan\left(\frac{\pi}{4} - x\right) = \tan 3x \to \frac{\pi}{4} - x = k\pi + 3x \to x = \frac{k\pi}{4} + \frac{\pi}{16}$$

(۹

$\sqrt{\cancel{\sin^2 x + \cos^2 x + 2\sin x\cos x}} = \sqrt{2}\,(\cos x - \sin x)(\cos x + \sin x)$

$\cancel{(\cos x + \sin x)} = \sqrt{2}\,(\cos x - \sin x)\cancel{(\cos x + \sin x)}$

$1 = \sqrt{2}\,(\cos x - \sin x) \Rightarrow 1 = -\sqrt{2}\sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$

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$-\frac{1}{2} = \sin\left(x - \frac{\pi}{4}\right) \Rightarrow$ ۲۱۰  ۳۳۰

$\Rightarrow x - \frac{\pi}{4} = 2k\pi - \frac{\pi}{3} \Rightarrow x = 2k\pi - \frac{\pi}{3} + \frac{\pi}{4}$

$x - \frac{\pi}{4} = 2k\pi + \cancel{\frac{\pi}{3}} + \frac{4\pi}{3} \qquad x = 2k\pi + \frac{4\pi}{3} + \frac{\pi}{4}$

$\sin 2x = -1 \rightarrow x = k\pi - \frac{\pi}{4} \rightarrow \frac{3\pi}{4}$

$x = k\pi + \frac{\pi}{12} \rightarrow \frac{\pi}{12}$

جواب
$\begin{cases} x = 2k\pi - \frac{\pi}{12} \\ x = 2k\pi + \frac{19\pi}{12} \end{cases}$

(۱۰

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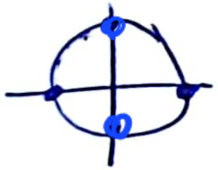
الف) در بازه $[0, 2\pi]$

$(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) = 1$



$\Rightarrow x = k\pi + \frac{\pi}{2}$ $\qquad x = \frac{3\pi}{2}, \frac{\pi}{2}$

ب)

$\sin^3 x - \cos^3 x = -1$

$\Rightarrow x = 2k\pi - \frac{3\pi}{2}$ $\qquad x = 0, \frac{3\pi}{2}, 2\pi$