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نام و نام خانوادگی .... پاسخنامه تشریحی تکلیف شماره ...۱۶... کلاس ....

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الف) $1-2\sin^2 x = 3\sin x - 1 \Rightarrow 2\sin^2 x + 3\sin x - 2 = 0 \Rightarrow (\sin x + 2)(2\sin x - 1) = 0$

$\sin x = -\frac{4}{2}$ غ ق ق / $\sin x = \frac{1}{2} \Rightarrow \sin x = \sin\frac{\pi}{6} \Rightarrow x = 2k\pi + \frac{\pi}{6}$, $x = 2k\pi + \frac{5\pi}{6}$

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ب) $2\cos^2 x - 1 + \cos x - 2 = 0 \Rightarrow 2\cos^2 x + \cos x - 3 = 0 \to (2\cos x + 3)(\cos x - 1) = 0$

$\cos x = -\frac{3}{2}$ غ ق ق $\vee$ $\cos x = 1 \to \cos x = \cos\frac{\pi}{2}$ $x = 2k\pi$

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الف) $(\underbrace{\sin^2 x - \cos^2 x}_{-\cos 2x})(\underbrace{\sin^2 x + \cos^2 x}_{1}) = \sin 4x \to -\cos 2x = 2\sin 2x\cos 2x$

$\longrightarrow (2\sin 2x + 1)(\cos 2x) = 0$

$\begin{cases} \cos 2x = 0 \to 2x = k\pi + \frac{\pi}{2} \to x = \frac{k\pi}{2} + \frac{\pi}{4} \\ \sin 2x = -\frac{1}{2} \Rightarrow \sin 2x = -\frac{\pi}{6} \longrightarrow 2x = 2k\pi - \frac{\pi}{6} \to x = k\pi - \frac{\pi}{12} \ \vee\ 2x = 2k\pi + \frac{7\pi}{6} \to x = k\pi + \frac{7\pi}{12} \end{cases}$

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ب) $2\cos^2 x - 1 + 2\sin x\cos x = 0 \Rightarrow \cos 2x + \sin 2x = 0 \to \underbrace{\cos 2x = -\sin 2x}_{\text{ربع دوم و چهارم}}$

$\longrightarrow 2x = 2k\pi - \frac{\pi}{4} \to x = \frac{k\pi}{2} - \frac{\pi}{8}$

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الف) $\frac{\cos x}{\sin x}(2\cos x + 1) = \frac{1}{\sin x} \xRightarrow{\sin x \neq 0} 2\cos^2 x + \cos x - 1 = 0 \begin{cases} \cos x = -1 \to \text{غ ق ق} \\ \cos x = \frac{1}{2} \to x = 2k\pi \pm \frac{\pi}{3} \end{cases}$

$\longrightarrow x = 2k\pi \pm \frac{\pi}{3}$

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ب) $\cancel{2\sin^2 x} + \cos^2 x - \sin x\cos x = \cancel{2\sin^2 x} + 2\cos^2 x \Rightarrow \cos^2 x + \cos x\sin x = 0$

$\Rightarrow \cos x(\cos x + \sin x) = 0 \Rightarrow \begin{cases} \cos x = 0 \longrightarrow x = k\pi + \frac{\pi}{2} \\ \cos x = -\sin x \xrightarrow{\text{ناحیه دوم و چهارم}} x = k\pi - \frac{\pi}{4} \end{cases}$

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الف) $\cos(\frac{\pi}{4} + x)\cos(\frac{\pi}{4} - x) = \frac{1}{4} \Rightarrow \cos(\frac{\pi}{4} + x)\sin(\frac{\pi}{4} + x) = \frac{1}{4} \to \sin(2x + \frac{\pi}{2}) = \frac{1}{2}$

$\to \cos 2x = \frac{1}{2} = \cos\frac{\pi}{3} \longrightarrow \begin{cases} 2x = 2k\pi + \frac{\pi}{3} \to x = k\pi + \frac{\pi}{6} \\ 2x = 2k\pi - \frac{\pi}{3} \to x = k\pi - \frac{\pi}{6} \end{cases}$

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ب) $\cos(2x - \frac{\pi}{6}) = \cos(\frac{\pi}{2} + 4x) \to \cancel{2x} - \frac{\pi}{6} = 2k\pi + \frac{\pi}{2} + \cancel{4x} \to 2k\pi + \frac{11\pi}{18} = 0$ غ ق ق

$\vee\ 2x - \frac{\pi}{6} = 2k\pi - \frac{\pi}{2} - 4x \to 6x = 2k\pi + -\frac{7\pi}{18} \to x = \frac{k\pi}{3} - \frac{7\pi}{72}$

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$\frac{2\cancel{\sin 2x}\cos 2x + \cancel{\sin 2x}}{\sin 2x} = 1 \xRightarrow{\sin 2x \neq 0 \to x \neq \frac{k\pi}{2}} 2\cos 2x + 1 = 1 \Rightarrow 2\cos 2x = 0$

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$\cos 2x = 0 \to 2x = k\pi + \frac{\pi}{2}$ $\quad x = \frac{k\pi}{2} + \frac{\pi}{4}$ $\Big\}\ x = \{\frac{k\pi}{2} + \frac{\pi}{4}\} - \{\ \}$

$\sin 2x \neq 0 \to x \neq \frac{k\pi}{2}$

$k=0 \to x=\pi$ $\quad k=-1 \to x=-\pi$ $\quad |-\pi-\pi| = 2\pi$

$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2} \Rightarrow \frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{\cos\frac{x}{2}+1}{\sin\frac{x}{2}} \Rightarrow$ $\Rightarrow \sin^2\frac{x}{2} = \cos^2\frac{x}{2} + 2\cos\frac{x}{2} + 1 \Rightarrow 1 - \cos^2\frac{x}{2} = 2\cos\frac{x}{2} + 1 \Rightarrow$ $\cancel{1} - 2\cos^2\frac{x}{2} = 2\cos\frac{x}{2} + \cancel{1} \Rightarrow 4\cos\frac{x}{2} = 0 \Rightarrow \cos\frac{x}{2} = 0 \Rightarrow \frac{x}{2} = k\pi + \frac{\pi}{2}$ $\Rightarrow x = 2k\pi + \pi$ ۱٫۵	
$\cos^2 x - \sin^2 x\cos^2 x = 1 \Rightarrow -\sin^2 x\cos^2 x = \sin^2 x \Rightarrow$ $\Rightarrow \sin^2 x(1+\cos^2 x) = 0 \Rightarrow \sin x = 0$ $\Rightarrow x = k\pi \to 0, \pi, 2\pi$ $\cos 3x = -1 \to 3x = 2k\pi + \pi \to x = \frac{2k\pi}{3} + \frac{\pi}{3} \to x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ ۵ جواب ۱٫۵	۷
$\tan\left(\frac{\pi}{4} - x\right) = \tan 3x \to \frac{\pi}{4} - x = k\pi + 3x \to 4x = \frac{\pi}{4} - k\pi$ $\to x = \frac{\pi}{16} - \frac{k\pi}{4}$ ۱٫۵ $\tan\left(\frac{\pi}{4} - x\right) = \tan 3x \to \frac{\pi}{4} - x = k\pi + 3x \to x = \frac{k\pi}{4} + \frac{\pi}{16}$	۸
$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \Rightarrow \sqrt{2}\cos x = \sqrt{2}\cos 2x$ $2\cos^2 x - 1$ $\Rightarrow x = 2x \to x = 0 \quad \cos x = k\pi + \frac{\pi}{2}$ $\sin 2x = -1 \to 2x = k\pi - \frac{2\pi}{4} \to \frac{3\pi}{4}$ $\to \frac{\pi}{2}$ یک جواب $\quad x = k\pi + \frac{\pi}{12} \to \frac{\pi}{12}$ ۱ ۲ جواب	۹
الف) $\sin^2 x - \cos^2 x = 1 \to \cos 2x = -1 = \cos\pi \to 2x = 2k\pi + \pi \to x = k\pi + \frac{\pi}{2}$ $2x = 2k\pi - \pi \to x = k\pi - \frac{\pi}{2}$ $k=0 \to \frac{\pi}{2} \quad k=1 \to \frac{3\pi}{2} \quad \to \frac{\pi}{2}, \frac{3\pi}{2}$ $k=1 \to \frac{\pi}{2} \quad k=2 \to \frac{3\pi}{2}$ ب) نقاط مرزی را امتحان کنیم $k=0 \to -1$ ✓ $\quad k=\pi \to 1$ ✗ $k=\frac{\pi}{2} \to 1$ ✗ $\quad k=\frac{3\pi}{2} \to -1$ ✓ $k = 0, \frac{3\pi}{2}, 2\pi$ ۲	۱۰