

الف) $1 - 2\sin^2 x = 3\sin x - 1 \rightarrow 2\sin^2 x + 3\sin x - 2 = 0 \rightarrow \begin{cases} \sin x = -\frac{1}{2} \rightarrow x = 2k\pi + \frac{3\pi}{2} \\ \sin x = -\frac{2}{3} \end{cases}$



ب) $2\cos^2 x - 1 + \cos x - 2 = 0 \rightarrow 2\cos^2 x + \cos x - 3 = 0 \rightarrow \begin{cases} \cos x = 1 \rightarrow x = 2k\pi \\ \cos x = -\frac{3}{2} \end{cases}$



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الف) $-\cos^2 x = \sin^2 x \rightarrow \cos(2x - \frac{\pi}{2}) = \cos(\frac{\pi}{2} - 2x)$
 $2x - \frac{\pi}{2} = 2k\pi + \frac{\pi}{2} - 2x \rightarrow 4x = 2k\pi + \pi \rightarrow x = k\pi + \frac{\pi}{2}$
 $2x - \frac{\pi}{2} = 2k\pi - \frac{\pi}{2} + 2x \rightarrow 0 = 2k\pi \rightarrow x = k\pi$
 ب) $2\sin x \cos x = 1 - 2\cos^2 x \rightarrow \sin 2x = -\cos 2x \rightarrow \cos(\frac{\pi}{2} - 2x) = \cos(2x)$
 $\frac{\pi}{2} - 2x = 2k\pi + 2x \rightarrow -\frac{\pi}{2} = 4k\pi \rightarrow x = -\frac{k\pi}{2} + \frac{\pi}{4}$
 $\frac{\pi}{2} - 2x = 2k\pi - 2x \rightarrow 0 = 2k\pi \rightarrow x = k\pi$

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الف) $\frac{\cos x}{\sin x} (2\cos x + 1) = \frac{1}{\sin x} \rightarrow 2\cos^2 x + \cos x = 0 \quad (\sin x \neq 0) \rightarrow \begin{cases} \cos x = 0 \rightarrow x = 2k\pi + \frac{\pi}{2} \\ \cos x = -\frac{1}{2} \rightarrow x = 2k\pi + \frac{2\pi}{3} \end{cases}$



ب) $\sin^2 x - \sin x \cos x = 1 \rightarrow -\cos^2 x = \sin x \cos x \rightarrow -\cos x = \sin x \rightarrow \cos(2x - \frac{\pi}{2}) = \cos(\frac{\pi}{2} - 2x)$
 $2x - \frac{\pi}{2} = 2k\pi + \frac{\pi}{2} - 2x \rightarrow -\frac{\pi}{2} = 4k\pi \rightarrow x = -\frac{k\pi}{2} + \frac{\pi}{4}$
 $2x - \frac{\pi}{2} = 2k\pi - \frac{\pi}{2} + 2x \rightarrow 0 = 2k\pi \rightarrow x = k\pi$

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الف) $\cos(\frac{\pi}{2} + x) \cos(x - \frac{\pi}{2}) = -\cos(\frac{\pi}{2} + x) \cos(\frac{\pi}{2} - x) = -\cos(\frac{\pi}{2} + x) \sin(\frac{\pi}{2} + x)$
 $= -\frac{1}{2} \sin(\frac{\pi}{2} + 2x) = -\frac{1}{2} \sin 2x = \frac{1}{2} \rightarrow \sin 2x = -\frac{1}{2} \rightarrow \begin{cases} 2x = 2k\pi + \frac{7\pi}{6} \rightarrow x = k\pi + \frac{7\pi}{12} \\ 2x = 2k\pi + \frac{11\pi}{6} \rightarrow x = k\pi + \frac{11\pi}{12} \end{cases}$

ب) $\cos(2x - \frac{\pi}{2}) = -\cos(\frac{\pi}{2} - 2x) = \cos(\frac{\pi}{2} + 2x)$
 $2x - \frac{\pi}{2} = 2k\pi + \frac{\pi}{2} + 2x \rightarrow -\frac{\pi}{2} = 4k\pi \rightarrow x = -\frac{k\pi}{2} + \frac{\pi}{4}$
 $2x - \frac{\pi}{2} = 2k\pi - \frac{\pi}{2} + 2x \rightarrow 0 = 2k\pi \rightarrow x = k\pi$

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الف) $1 + \frac{2\sin^2 x \cos^2 x}{\sin 2x} = \cos^2 x + \sin^2 x = 1 \rightarrow \cos^2 x = 0 \quad (\sin 2x \neq 0)$

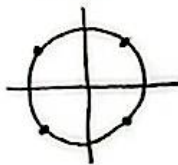
$2x = 2k\pi + \frac{\pi}{2} \rightarrow x = k\pi + \frac{\pi}{4}$
 $2x = 2k\pi - \frac{\pi}{2} \rightarrow x = k\pi - \frac{\pi}{4}$



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$$\frac{\sin \frac{x}{y}}{1 + \cos \frac{x}{y}} = \frac{1 + \cos \frac{x}{y}}{\sin \frac{x}{y}} \rightarrow \tan x = \frac{1}{\tan x} \rightarrow \tan^2 x = 1 \quad (\tan x \neq 0)$$

$$\tan x = 1 \rightarrow x = k\pi + \frac{\pi}{4}$$



$$\left| k\pi + \frac{\pi}{4} - k\pi - \frac{\pi}{4} \right| = \frac{\pi}{4}$$

$$\tan x = -1 \rightarrow x = k\pi + \frac{3\pi}{4}$$

$$x \in [-\pi, \frac{3\pi}{4}] \rightarrow x = \left\{ -\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4} \right\}$$

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$$1 - \sin^2 x - \sin^2 x \cos^2 x = 1 \rightarrow \sin^2 x (1 + \cos^2 x) = 0$$

$$\sin^2 x = 0 \rightarrow \sin x = 0 \rightarrow x = k\pi$$

$$1 + \cos^2 x = 0 \rightarrow \cos^2 x = -1 \rightarrow x = 2k\pi + \pi \rightarrow x = \frac{2k\pi}{2} + \frac{\pi}{2}$$

$$x \in [0, 2\pi] \rightarrow x = \left\{ 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi \right\}$$

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$$\tan \left(x - \frac{\pi}{4} \right) = \tan^2 x \rightarrow 2x = k\pi - \frac{\pi}{4} + x \rightarrow x = \frac{k\pi}{2} - \frac{\pi}{4}$$

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$$1 + \sin^2 x = 2 \cos^2 x \rightarrow \sin^2 x = \cos^2 x \rightarrow \cos^2 x = \cos^2 \left(\frac{\pi}{2} - x \right)$$

$$2x = 2k\pi + \frac{\pi}{2} - 2x \rightarrow 4x = 2k\pi + \frac{\pi}{2} \rightarrow x = \frac{k\pi}{2} + \frac{\pi}{4}$$

$$2x = 2k\pi - \frac{\pi}{2} + 2x \rightarrow 2x = 2k\pi - \frac{\pi}{2} \rightarrow x = k\pi - \frac{\pi}{4}$$

$$x \in [0, 2\pi] \rightarrow x = \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$$

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$$\text{الف) } -\cos^2 x = 1 \rightarrow \cos^2 x = -1 \rightarrow 2x = 2k\pi + \frac{\pi}{2} \rightarrow x = k\pi + \frac{\pi}{4}$$

$$x \in [0, 2\pi] \rightarrow x = \left\{ \frac{\pi}{4}, \frac{5\pi}{4} \right\}$$



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$$\text{ب) } (\sin x - \cos x) (\sin^2 x + \cos^2 x + \sin x \cos x) = 0$$

$$\rightarrow 1 + \frac{1}{2} \sin^2 x = \cos x - \sin x \rightarrow \frac{1}{2} \sin^2 x + \sin^2 x + \sin x \cos x = 1 - \sin^2 x$$

$$\rightarrow \sin^2 x + \frac{1}{2} \sin^2 x = 0 \rightarrow \begin{cases} \sin^2 x = 0 \rightarrow 2x = k\pi \rightarrow x = \frac{k\pi}{2} \\ \sin^2 x = -\frac{1}{2} \text{ (no real solution)} \end{cases}$$

$$x \in [0, 2\pi] \rightarrow x = \left\{ 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2} \right\}$$

