علیرضا تجلی :

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① (I) $\cos 2x = 2\cos^2 x - 1$

(II) $\cos 2x = 1 - 2\sin^2 x$

الف $\to \cos 2x = 3\sin x - 1 \xrightarrow{(II)} 1 - 2\sin^2 x = 3\sin x - 1$

$\to 2\sin^2 x + 3\sin x - 2 = 0 \to (2\sin x - 1)(\sin x + 2)$

یا $\boxed{\sin x = -2 \text{ غ‌ق‌ق}}$

$\boxed{\sin x = \frac{1}{2}} \to \sin x = \sin\frac{\pi}{6}$

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$$x = \begin{cases} 2k\pi + \frac{\pi}{6} \to 2k\pi + \frac{\pi}{6} \\ 2k\pi + \pi - \frac{\pi}{6} \to 2k\pi + \frac{5\pi}{6} \end{cases}$$

ب $\to \cos 2x + \cos x - 2 = 0 \xrightarrow{(I)} 2\cos^2 x + \cos x - 3 = 0$

$(2\cos x + 3)(\cos x - 1)$

$\cos x = -\frac{3}{2}$ غ‌ق‌ق یا $\qquad \cos x = 1 \to \cos x = \cos 0°$

جواب $\begin{cases} x = 2k\pi + 0 \\ x = 2k\pi - 0 \end{cases} \to \boxed{x = 2k\pi}$

② (I) $\cos 2x = \cos^2 x - \sin^2 x$

(II) $\cos 2x = \cos^4 x - \sin^4 x$

(III) $\sin 2x = 2\sin x \cos x$

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الف $\to \sin^4 x - \cos^4 x = \sin 4x \xrightarrow[(III)]{(II)} -\cancel{\cos 2x} = 2\sin 2x \cancel{\cos 2x}$

$\cos 2x = 0$ $\qquad x = \frac{k\pi}{2} + \frac{\pi}{4}$

$\cos 2x = \cos 0° \to \boxed{x = 2k\pi}$

$-\frac{1}{2} = \sin 2x$

$\sin 2x = \sin\frac{11\pi}{6}$

$$2x = \begin{cases} 2k\pi + \frac{11\pi}{6} \to \boxed{x = k\pi + \frac{11\pi}{12}} \\ 2k\pi + \pi - \frac{11\pi}{6} \to \boxed{x = k\pi - \frac{5\pi}{12}} \end{cases}$$

$\underbrace{\pi - \frac{11\pi}{6}}_{-\frac{5\pi}{6}}$

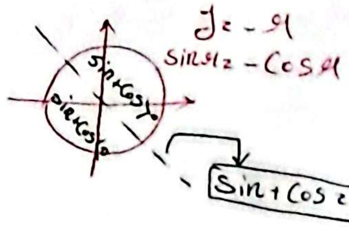
$\Rightarrow (2k\pi) \cup (k\pi + \frac{11\pi}{12}) \cup (k\pi - \frac{5\pi}{12})$

ب $\to 2\sin^2 x - \sin x\cos x + \cos^2 x = 2 \xrightarrow{*} 2\underbrace{(\sin^2 x - 1)}_{-\cos^2 x} - \sin x\cos x + \cos^2 x = 0$

$* \ \boxed{\cos^2 x = 1 - \sin^2 x}$

$-\cos^2 x - \sin x\cos x = 0 \to -\cos x(\cos x + \sin x) = 0$

$$\to \begin{cases} \cos x = 0 \to \boxed{x = 2k\pi} \\ \sin x = -\cos x \to \text{طبق شکل} \to \boxed{x = k\pi - \frac{\pi}{4}} \end{cases}$$




$\to x \in \{2k\pi \cup k\pi - \frac{\pi}{4}\}$

* سوال ۲ و ۳ قسمت ج را بلد نبودم *

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الف $\to \underbrace{\cot x}_{\frac{\cos x}{\sin x}}(2\cos x+1)=\frac{1}{\sin x} \to 2\cos^2 x+\cos x=1 \to \underbrace{2\cos^2 x-1}_{\cos 2x}=-\cos x$

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$\cos 2x=-\underbrace{\cos x}_{\cos(x+\pi)}$

$\cos 2x=\cos(\pi+x) \to \begin{cases} 2x=2k\pi+\pi+x \to \boxed{x=2k\pi+\pi \text{ غ}} \\ 3x:\ 2x=2k\pi-\pi-x \to x=\frac{2k\pi}{3}-\frac{\pi}{3} \end{cases}$

چون sin آن صفر می‌شود که در رابطه بالا نمی‌تواند باشد پس

$\to x\in \frac{2k\pi}{\cancel{3}}+\frac{\pi}{3}$

ب $\to 2\cos^2 x+2\sin x\cos x=1 \xrightarrow[2\cos^2 x-1=\cos 2x]{2\sin x\cos x=\sin 2x} \underbrace{-\cos 2x}_{\sin(\frac{3\pi}{2}+2x)}=\sin 2x$

$\longrightarrow \sin 2x=\sin(\frac{3\pi}{2}+2x)$

$\longrightarrow 2x \begin{cases} 2k\pi+\frac{3\pi}{2}+2x & \text{جواب ندارد} \\ 2k\pi-\frac{3\pi}{2}-2x+\pi \longrightarrow 4x=2k\pi-\frac{\pi}{2} \to \boxed{x=\frac{k\pi}{2}-\frac{\pi}{8}} \end{cases}$

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الف $\to \cos(\underbrace{\frac{\pi}{4}+x}_{\text{متمم با } \frac{\pi}{4}-x})\cos(x-\frac{\pi}{4})=\frac{1}{4} \longrightarrow \underbrace{\sin(\frac{\pi}{4}-x)\cos(\frac{\pi}{4}-x)}=\frac{1}{4}$

می‌توان در - ضرب کرد چون $\cos x=\cos -x$

$\frac{1}{\cancel{2}}\sin(\frac{\pi}{2}-2x)=\frac{\cancel{1}}{\cancel{4}}\frac{1}{2}$

$\cos 2x=\cancel{\frac{1}{2}}\cos\frac{\pi}{3}$

$2x=2k\pi\pm\frac{\pi}{3} \to \begin{cases} x=k\pi+\frac{\pi}{6} \\ x=k\pi-\frac{\pi}{6} \end{cases}$

ب

ب $\to \cancel{\cos(2x-\frac{\pi}{9})}=\cancel{-\sin 2x}$

$\boxed{\sin(\frac{\pi}{2}+x)=\cos x}$

$\boxed{-\sin x=\sin -x}$

$\sin(\underbrace{\frac{\pi}{2}+2x-\frac{\pi}{9}}_{2x+\frac{7\pi}{18}})=\sin(-2x)$

$\to -2x=2k\pi+2x+\frac{7\pi}{18} \longrightarrow -\cancel{4}x=\frac{\cancel{2}k\pi}{2}+\frac{7\pi}{\cancel{18}}-\frac{7\pi}{72}$

$\to \boxed{x=\frac{k\pi}{2}-\frac{7\pi}{72}}$

۵ $\boxed{\sin 2x=2\sin x\cos x}$

$\frac{\overbrace{\sin 4x}^{2\sin 2x\cos 2x}+\sin 2x}{\sin 2x}-\sin^2 x=\cos^2 x \to \frac{(2\cos 2x+1)\cancel{\sin 2x}}{\cancel{\sin 2x}}-\sin^2 x=\cos^2 x$

$\to 2\cos 2x+1=1 \to 2\cos 2x=0 \to \cos 2x=0 \to \cos\frac{\pi}{2}$

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$2x \to \{2k\pi\pm\frac{\pi}{2} \to \boxed{x=\frac{k\pi}{2}\pm\frac{\pi}{4}}$

(۶)

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \underbrace{\frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2}}_{\frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}} \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} \to \underbrace{\left(\sin\frac{x}{2}\right)^2}_{1-\cos^2\frac{x}{2}} = 1+\cos^2\frac{x}{2}+2\cos\frac{x}{2}$$

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$$2\cos^2\frac{x}{2}+2\cos\frac{x}{2}=0 \to 2\cos\frac{x}{2}\left(\cos\frac{x}{2}+1\right)=0$$

$$\cos\frac{x}{2}=0 = \cos\frac{\pi}{2}$$

$$\frac{x}{2} \to 2k\pi \pm \frac{\pi}{2}$$

$$\to x = 4k\pi \pm \pi \to x = 2k\pi \pm \pi$$

$2k\pi$

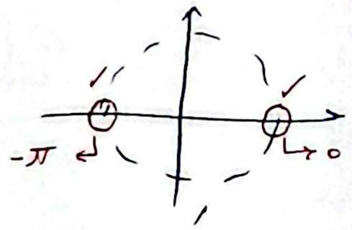
$$\cos\frac{x}{2} = -1 = \cos\pi$$

$$\to \frac{x}{2} = 2k\pi \pm \pi$$

$$\to x = 4k\pi \pm 2\pi$$

$2k\pi$

$$\to x = 2k\pi \pm 2\pi = 2k\pi$$




$|-\pi-\pi| = 2\pi$

$\to |-\pi-0| = \pi$

(۷)

$$-\sin^2 x\cos 3x = (1-\cos^2 x)\sin^2 x$$

$$\sin x = 0 \to \boxed{x = k\pi}$$

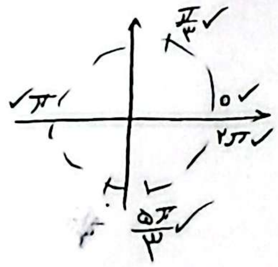
$$-\cos 3x = 1$$

$$\cos 3x = \cos\pi$$

$$3x = 2k\pi \pm \pi$$

$$\to x = \frac{2k\pi}{3} \pm \frac{\pi}{3}$$

(۲)




۰ و ۲π ۲ تا حساب بشه

$\to$ تعداد جواب: ۵ تا

(۸)

$$\frac{1+\tan x}{1-\tan x} = \tan\left(\frac{\pi}{4}+x\right)$$

$$\frac{1-\tan x}{1+\tan x} = \tan\left(\frac{\pi}{4}-x\right)$$

$$\frac{1-\tan x}{1+\tan x} = \tan\left(\frac{\pi}{4}-x\right) \to \tan\left(\frac{\pi}{4}-x\right) = \tan 3x$$

(۲)

$$\to 3x = k\pi + \frac{\pi}{4} - x \to 4x = k\pi + \frac{\pi}{4} \to x = \frac{k\pi}{4} + \frac{\pi}{16}$$

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$1+\sin 2x \to \sin^2+\cos^2+2\sin\cos \to (\sin+\cos)^2$

$a\sin x+b\cos x=\frac{a}{|a|}\sqrt{a^2+b^2}\sin(x+\alpha)$

$\tan\alpha=\frac{b}{a}$

$\sqrt{1+\sin 2x}=\sqrt{2}\cos 2x$

$|\sin+\cos|=\sqrt{2}\cos 2x$

$|\sqrt{2}\sin(x+\frac{\pi}{4})|=\sqrt{2}\cos 2x$

1 $\to \cancel{\sqrt{2}}\sin(x+\frac{\pi}{4})=\cancel{\sqrt{2}}\sin(\frac{\pi}{2}+2x)$ 1,5

2 $\to -\sqrt{2} \ldots = \ldots$ ۲جواب

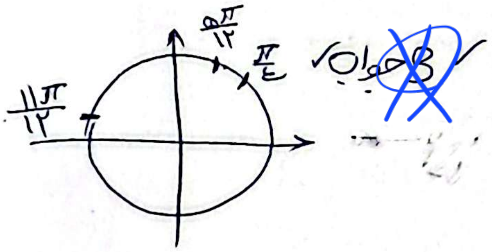
① $\to x+\frac{\pi}{4} \to 2k\pi+\frac{\pi}{2}+2x \to -x=2k\pi+\frac{\pi}{4} \to \boxed{x=2k\pi-\frac{\pi}{4}}$

$\to 2k\pi+\pi-\frac{\pi}{2}-2x \to 3x=2k\pi+\frac{3\pi}{4} \to \boxed{x=\frac{2k\pi}{3}+\frac{\pi}{4}}$

② $-\underbrace{\sin(x+\frac{\pi}{4})}_{\sin(-x-\frac{\pi}{4})}=\sin(\frac{\pi}{2}+2x) \to -x-\frac{\pi}{4}=2k\pi+\frac{\pi}{2}+2x$

$\to -3x=2k\pi+\frac{3\pi}{4} \to \boxed{x=\frac{2k\pi}{3}-\frac{\pi}{4}}$

$\to -x-\frac{\pi}{4}=2k\pi+\pi-\frac{\pi}{2}-2x$




$\to \boxed{x=2k\pi+\frac{\pi}{4}}$

$\sin 2x=-1 \to x = k\pi-\frac{\pi}{4} \to \frac{3\pi}{4}$

$x=k\pi+\frac{\pi}{12} \to \frac{\pi}{12}$

$\cos^4 x\,\sin^4 x\,\cos 2x$ 10

الف $\to 4\cos 2x = 4(-1) \to 2x=2k\pi\pm\pi$

$\cos\pi \to \boxed{x=k\pi\pm\frac{\pi}{2}} \to \frac{\pi}{2},\frac{3\pi}{2}$ 1,5

ب $\to \sin^3 x-\cos^3 x=-1 \to \underbrace{(\sin x-\cos x)}_{\sqrt{1-\sin x}}(\underbrace{\sin^2 x+\cos^2 x}_{1}+\sin x\cos x)=-1$

$\to \sqrt{1-\sin 2x}\,(1+\frac{1}{2}\sin 2x)=-1$

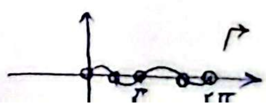
$\to (1+\frac{1}{2}\sin 2x)^2=\left(-\frac{1}{\sqrt{1-\sin 2x}}\right)^2 \xrightarrow{\text{توان ۲}} 1+\frac{1}{4}\sin^2 2x+\sin 2x=\frac{1}{1-\sin 2x}$

$\to \cancel{1}-\cancel{\sin 2x}+\frac{1}{4}\sin^3 2x+\frac{1}{4}\sin^2 2x+\cancel{\sin 2x}-\sin^2 2x=\cancel{1}$

$\frac{1}{4}\sin^3 2x-\frac{3}{4}\sin^2 2x=0 \to \frac{1}{4}\sin^2 2x(\sin 2x-3)=0$

غ‌ق‌ق ۳ ها

$\sin 2x=0 \to$



$\{\cancel{\frac{\pi}{2}},\cancel{\pi},\frac{3\pi}{2},2\pi\}$ 0