علیرضا تجلی :

① (I) $\cos 2x = 2\cos^2 x - 1$

(II) $\cos 2x = 1 - 2\sin^2 x$

الف $\rightarrow \underline{\cos 2x = 3\sin x - 1} \xrightarrow{(II)} 1 - 2\sin^2 x = 3\sin x - 1$

$\longrightarrow 2\sin^2 x + 3\sin x - 2 = 0 \longrightarrow (2\sin x - 1)(\sin x + 2)$

یا $\boxed{\sin x = -2 \text{ غ‌ق‌ق}}$

$\boxed{\sin x = \frac{1}{2}} \rightarrow \sin x = \sin\frac{\pi}{6}$

$$x = \begin{cases} 2k\pi + \frac{\pi}{6} \rightarrow 2k\pi + \frac{\pi}{6} \\ 2k\pi + \pi - \frac{\pi}{6} \rightarrow 2k\pi + \frac{5\pi}{6} \end{cases}$$

ب $\rightarrow \cos 2x + \cos x - 2 = 0 \xrightarrow{(I)} 2\cos^2 x + \cos x - 3 = 0$

$(2\cos x + 3)(\cos x - 1)$

$\cos x = -\frac{3}{2}$ غ‌ق‌ق

یا $\underline{\cos x = 1} \rightarrow \cos x = \cos 0^\circ$

جواب $\begin{cases} x = 2k\pi + 0 \\ x = 2k\pi - 0 \end{cases} \rightarrow \boxed{x = 2k\pi}$

② (I) $\cos 2x = \cos^2 x - \sin^2 x$

(II) $\cos 2x = \cos^4 x - \sin^4 x$

(III) $\sin 2x = 2\sin x \cos x$

الف $\rightarrow \sin^4 x - \cos^4 x = \sin 4x \xrightarrow[(III)]{(II)} -\cancel{\cos 2x} = 2\sin 2x \cancel{\cos 2x}$

$\leftarrow \cos 2x = 0$

$\leftarrow -\frac{1}{2} = \sin 2x$

$\cos 2x = \cos 0^\circ \rightarrow \boxed{x = 2k\pi}$

$\sin 2x = \sin\frac{11\pi}{6}$

یا $2x = \begin{cases} 2k\pi + \frac{11\pi}{6} \rightarrow \boxed{x = k\pi + \frac{11\pi}{12}} \\ 2k\pi + \underbrace{\pi - \frac{11\pi}{6}}_{\frac{-5\pi}{6}} \rightarrow \boxed{x = k\pi - \frac{5\pi}{12}} \end{cases}$

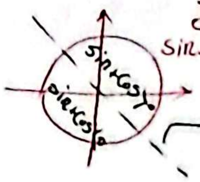
$\Rightarrow (2k\pi) \cup (k\pi + \frac{11\pi}{12}) \cup (k\pi - \frac{5\pi}{12})$

ب $\rightarrow 2\sin^2 x - \sin x\cos x + \cos^2 x = 2 \xrightarrow{*} \underbrace{2(\sin^2 x - 1)}_{-\cos^2 x} - \sin x\cos x + \cos^2 x = 0$

$\boxed{* \cos^2 x = 1 - \sin^2 x}$

$-\cos^2 x - \sin x\cos x = 0 \rightarrow -\cos x(\cos x + \sin x) = 0$

$\rightarrow \begin{cases} \cos x = 0 \rightarrow \boxed{x = 2k\pi} \\ \sin x = -\cos x \rightarrow \text{طبق شکل} \rightarrow \boxed{x = k\pi - \frac{\pi}{4}} \end{cases}$



$y = -x$

$\sin x = -\cos x$

$\boxed{\sin + \cos = 0}$

$\rightarrow x \in \{2k\pi \cup k\pi - \frac{\pi}{4}\}$

* سوال ۲ و ۳ قسمت ج را جابجا نوشتم *

(۳)

الف $\to \underbrace{\cot x}_{\frac{\cos x}{\sin x}}(2\cos x+1)=\frac{1}{\sin x} \to 2\cos^2 x+\cos x=1 \to \underbrace{2\cos^2 x-1}_{\cos 2x}=-\cos x$

$\cos 2x=\underbrace{-\cos x}_{\cos(x+\pi)}$

$$\cos 2x=\cos(\pi+x) \to \begin{cases} 2x=2k\pi+\pi+x \to \boxed{x=2k\pi+\pi} \\ 2x=2k\pi-\pi-x \to x=\frac{2k\pi}{3}-\frac{\pi}{3} \end{cases}$$

چون sin آن صفر شود که در رابطه بالا نمی‌تواند باشد پس

$x\in \frac{2k\pi}{3}-\frac{\pi}{3}$

ب $\to 2\cos^2 x+2\sin x\cos x=1 \xrightarrow[2\cos^2 x-1=\cos 2x]{2\sin x\cos x=\sin 2x} \underbrace{-\cos 2x}_{\sin(\frac{3\pi}{2}+2x)}=\sin 2x$

$\to \sin 2x=\sin(\frac{3\pi}{2}+2x)$

$$\to 2x\begin{cases} 2k\pi+\frac{3\pi}{2}+2x \quad \text{جواب ندارد} \\ 2k\pi-\frac{3\pi}{2}-2x+\pi \to 4x=2k\pi-\frac{\pi}{2} \to \boxed{x=\frac{k\pi}{2}-\frac{\pi}{8}} \end{cases}$$

(۴)

الف $\to \cos(\frac{\pi}{4}+x)\cos(x-\frac{\pi}{4})=\frac{1}{4} \xrightarrow{} \sin(\frac{\pi}{4}-x)\cos(\frac{\pi}{4}-x)=\frac{1}{4}$

متمم با $\frac{\pi}{4}-x$

می‌توان در ۱- ضرب کرد چون $\cos x=\cos -x$

$\cancel{\frac{1}{2}}\sin(\frac{\pi}{2}-2x)=\cancel{\frac{1}{4}}\frac{1}{2}$

$\cos 2x=\cancel{\frac{1}{2}}\cos\frac{\pi}{3}$

$$2x=2k\pi\pm\frac{\pi}{3}\begin{cases} x=k\pi+\frac{\pi}{6} \\ x=k\pi-\frac{\pi}{6} \end{cases}$$

ب $\to \cancel{\cos(2x-\frac{\pi}{9})}=\cancel{-\sin 2x}$

$\sin(\underbrace{\frac{\pi}{2}+2x-\frac{\pi}{9}}_{2x+\frac{7\pi}{18}})=\sin(-2x)$

$\boxed{\sin(\frac{\pi}{2}+x)=\cos x}$

$\boxed{-\sin x=\sin -x}$

$\to -2x=2k\pi+2x+\frac{7\pi}{18} \to \cancel{-4}x=\frac{2k\pi}{4}+\frac{7\pi}{18}-\frac{7\pi}{72}$

$\to \boxed{x=\frac{k\pi}{2}-\frac{7\pi}{72}}$

(۵) $\boxed{\sin 2x=2\sin x\cos x}$

$\frac{\overbrace{\sin 4x}^{2\sin 2x\cos 2x}+\sin 2x}{\sin 2x}-\sin^2 x=\cos^2 x \to \frac{(2\cos 2x+1)\cancel{\sin 2x}}{\cancel{\sin 2x}}-\sin^2 x=\cos^2 x$

$\to 2\cos 2x+1=1 \to 2\cos 2x=0 \to \cos 2x=0 \to \cos\frac{\pi}{2}$

$2x \to \{2k\pi\pm\frac{\pi}{2} \to \boxed{x=k\pi\pm\frac{\pi}{4}}$

۶

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \underbrace{\frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2}}_{\frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}} \quad \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} \leadsto \underbrace{\left(\sin\frac{x}{2}\right)^2}_{1-\cos^2\frac{x}{2}} = 1+\cos^2\frac{x}{2}+2\cos\frac{x}{2}$$

$$2\cos^2\frac{x}{2} + 2\cos\frac{x}{2} = 0 \to 2\cos\frac{x}{2}\left(\cos\frac{x}{2}+1\right) = 0$$

$\leftarrow \cos\frac{x}{2} = 0 = \cos\frac{\pi}{2}$

$\frac{x}{2} \to 2k\pi \pm \frac{\pi}{2}$

$\to x = 4k\pi \pm \pi \to \underline{x = 2k\pi \pm \pi}$

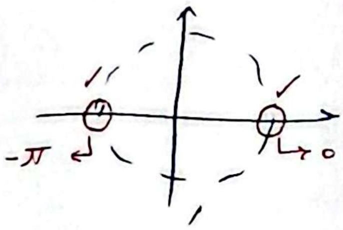
$4k\pi \to 2k\pi$

$\cos\frac{x}{2} = -1 = \cos\pi$

$\to \frac{x}{2} = 2k\pi \pm \pi$

$\to x = 4k\pi \pm 2\pi$ ($4k\pi \to 2k\pi$)

$\to x = 2k\pi \pm 2\pi = 2k\pi$



$\to |-\pi - 0| = \pi$

۷

$$-\sin^2 x\cos 3x = (1-\cos^2 x)\sin^2 x$$

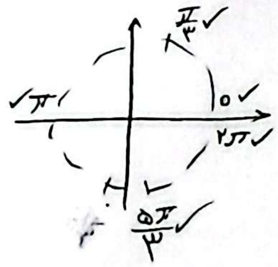
$\to \sin x = 0 \to \boxed{x = k\pi}$

$\to -\cos 3x = 1$

$\cos 3x = \cos\pi$

$3x = 2k\pi \pm \pi$

$\to x = \frac{2k\pi}{3} \pm \frac{\pi}{3}$



۰ و ۲π ۲تا حساب میشه

$\to$ تعداد جواب: ۵تا

۸

$\frac{1+\tan x}{1-\tan x} = \tan\left(\frac{\pi}{4}+x\right)$

$\frac{1-\tan x}{1+\tan x} = \tan\left(\frac{\pi}{4}-x\right)$

$$\frac{1-\tan x}{1+\tan x} = \tan\left(\frac{\pi}{4}-x\right) \to \tan\left(\frac{\pi}{4}-x\right) = \tan 3x$$

$$\to 3x = k\pi + \frac{\pi}{4} - x \to 4x = k\pi + \frac{\pi}{4} \to x = \frac{k\pi}{4} + \frac{\pi}{16}$$

$1+\sin 2x \to \sin^2+\cos^2+2\sin\cos \to (\sin+\cos)^2$ ⑨

$a\sin x + b\cos x = \frac{a}{|a|}\sqrt{a^2+b^2}\,\sin(x+\alpha)$

$\tan\alpha = \frac{b}{a}$

$\underbrace{\sqrt{1+\sin 2x}} = \sqrt{2}\,\underline{\cos 2x}$

$|\sin+\cos| = \sqrt{2}\cos 2x$

$|\sqrt{2}\sin(x+\frac{\pi}{4})| = \sqrt{2}\cos 2x$

$\xrightarrow{1} \cancel{\sqrt{2}}\sin(x+\frac{\pi}{4}) = \cancel{\sqrt{2}}\sin(\frac{\pi}{2}+2x)$

$\xrightarrow{2} -\sqrt{2}\ \text{''} = \text{''}$

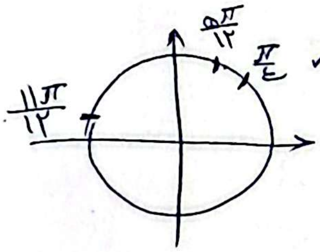
① $\to x+\frac{\pi}{4} \to 2k\pi+\frac{\pi}{2}+2x \to -x = 2k\pi+\frac{\pi}{4} \to \boxed{x = 2k\pi-\frac{\pi}{4}}$

$\to 2k\pi+\pi-\frac{\pi}{2}-2x \to 3x = 2k\pi+\frac{3\pi}{4} \to \boxed{x = \frac{2k\pi}{3}+\frac{\pi}{4}}$

② $\underbrace{-\sin(x+\frac{\pi}{4})}_{\sin(-x-\frac{\pi}{4})} = \sin(\frac{\pi}{2}+2x) \to -x-\frac{\pi}{4} = 2k\pi+\frac{\pi}{2}+2x$

$\to -3x = 2k\pi+\frac{3\pi}{4} \to \boxed{x = \frac{2k\pi}{3}-\frac{\pi}{4}}$

$\to -x-\frac{\pi}{4} = 2k\pi+\pi-\frac{\pi}{2}-2x$



جواب ✓

$\to \boxed{x = 2k\pi+\frac{\pi}{4}}$

$\cos^4 x\ \sin^4 x\ \cos 2x$ ⑩

الف $\to 4\cos 2x = 4(-1) \to 2x = 2k\pi \pm \pi$

$\cos\pi \to \boxed{x = k\pi \pm \frac{\pi}{2}} \to \frac{\pi}{2}, \frac{3\pi}{2}$

ب $\to \sin^3 x - \cos^3 x = -1 \to \underbrace{(\sin x - \cos x)}_{\sqrt{1-\sin x}}(\underbrace{\sin^2 x+\cos^2 x}_{1}+\sin x\cos x) = -1$

$\to \sqrt{1-\sin 2x}\,(1+\frac{1}{2}\sin 2x) = -1$

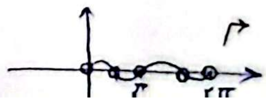
$\to (1+\frac{1}{2}\sin 2x)^2 = \left(-\frac{1}{\sqrt{1-\sin 2x}}\right)^2 \xrightarrow{\text{۲ توان}} 1+\frac{1}{4}\sin^2 2x+\sin 2x = \frac{1}{1-\sin 2x}$

$\to \cancel{1}-\cancel{\sin 2x}+\frac{1}{4}\sin^3 2x+\frac{1}{4}\sin^2 2x+\cancel{\sin 2x}-\sin^2 2x = \cancel{1}$

$\frac{1}{4}\sin^3 2x-\frac{3}{4}\sin^2 2x = 0 \to \frac{1}{4}\sin^2 2x\,(\underbrace{\sin 2x-3}_{0}) = 0$

غ ق ق ۳ ها

$\sin 2x = 0 \to$



$\{\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi\}$