نام و نام خانوادگی ..پوریا بلالیمانی........ پاسخنامه تشریحی تکلیف شماره ..۱.. کلاس دوازدهم ریاضی A..........

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الف) $\cos 2x = 3\sin x - 1 \to \underbrace{\cos^2 x}_{1-\sin^2 x} - \sin^2 x = 3\sin x - 1 \to 1 - 2\sin^2 x = 3\sin x - 1$

$\to 2\sin^2 x + 3\sin x - 2 = 0 \to \sin x = -2$ غ‌ق‌ق ، $\sin x = \frac{1}{2}$ ✓ $\to \sin x = \sin\frac{\pi}{6} \to \begin{cases} x = 2k\pi + \frac{\pi}{6} \\ x = 2k\pi + \pi - \frac{\pi}{6} = 2k\pi + \frac{5\pi}{6} \end{cases}$

ب) $\cos 2x + \cos x - 2 = 0 \to \cos^2 x - (1 - \cos^2 x) + \cos x - 2 = 0$

$\to 2\cos^2 x + \cos x - 3 = 0 \to \cos x = -\frac{3}{2}$ غ‌ق‌ق ،

$\cos x = 1$ ✓ $\to x = 2k\pi$

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الف) $\sin^4 x - \cos^4 x = \sin 4x \to -\cos 2x = \sin 4x \to \cos 2x = -\sin 4x = \cos(\frac{\pi}{2} + 4x)$

$\to 4x + \frac{\pi}{2} = 2k\pi + 2x \to 2x = 2k\pi - \frac{\pi}{2} \to x = k\pi - \frac{\pi}{4}$

$4x + \frac{\pi}{2} = 2k\pi - 2x \to 6x = 2k\pi - \frac{\pi}{2} \to x = \frac{k\pi}{3} - \frac{\pi}{12}$

ب) $2\cos^2 x + 2\sin x\cos x = \underbrace{1}_{\sin^2 x + \cos^2 x = 1} \to 2\cos^2 x + \underbrace{2\sin x\cos x}_{\sin 2x} = \sin^2 x + \cos^2 x$

$\to \sin 2x = -\cos 2x \to \tan 2x = -1 \to 2x = k\pi - \frac{\pi}{4} \to x = \frac{k\pi}{2} - \frac{\pi}{8}$

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الف) $\cot x(2\cos x + 1) = \frac{1}{\sin x} \to \cos x(2\cos x + 1) = 1 \to 2\cos^2 x + \cos x - 1 = 0$

$\to \cos x = 1$ غ‌ق‌ق ← چون $\sin x$ صفر می‌شود و مخرج ۰ می‌شود.

$\cos x = \frac{1}{2} \to x = 2k\pi \pm \frac{\pi}{3}$

ب) $2\sin^2 x - \sin x\cos x + \cos^2 x = 2 \to 2\sin^2 x - \sin x\cos x + \cos^2 x = 2\sin^2 x + 2\cos^2 x$

$\to -\sin x\cos x = \cos^2 x \to \cos x = -\sin x \to \cos x = \cos(\frac{\pi}{2} + x) \to$

$\frac{\pi}{2} + x = 2k\pi - x \to 2x = 2k\pi - \frac{\pi}{2} \to x = k\pi - \frac{\pi}{4}$

از آن جایی که $\cos x$ را از طرفین حذف کرده‌ایم پس باید عبارت $\cos x = 0$ را هم بررسی کنیم $\to x = 2k\pi \pm \frac{\pi}{2}$

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الف) $\cos(\frac{\pi}{4} + x)\underbrace{\cos(x - \frac{\pi}{4})}_{\cos(\frac{\pi}{4} - x)} = \frac{1}{4} \to \cos(\frac{\pi}{4} + x)\sin(\frac{\pi}{4} + x) = \frac{1}{4}$

$\to \sin(2x + \frac{\pi}{2}) = \frac{1}{2} \to \cos 2x = \frac{1}{2}$

$\to 2x = 2k\pi \pm \frac{\pi}{3} \to x = k\pi \pm \frac{\pi}{6}$

ب) $\cos(2x - \frac{\pi}{9}) = -\sin 2x \to \cos(2x - \frac{\pi}{9}) = \cos(\frac{\pi}{2} + 2x)$

$\to 2x - \frac{\pi}{9} = 2k\pi + \frac{\pi}{2} + 2x$ ✗

$\to 2x - \frac{\pi}{9} = 2k\pi - \frac{\pi}{2} - 2x \to 4x = 2k\pi - \frac{7\pi}{18} \to x = \frac{k\pi}{2} - \frac{7\pi}{72}$

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$\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x \to \frac{\sin 2x(2\cos 2x + 1)}{\sin 2x} = \cos^2 x + \sin^2 x \to 2\cos 2x = 0$

$\to 2x = 2k\pi \pm \frac{\pi}{2} \to x = k\pi \pm \frac{\pi}{4}$

$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2} \to \frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}} \to \tan\frac{x}{4} = \cot\frac{x}{4}$

$\to \tan^2\frac{x}{4}=1 \to \tan\frac{x}{4}=\pm 1 \to \frac{x}{4}=k\pi\pm\frac{\pi}{4} \to x=4k\pi+\pi \xrightarrow{\text{جواب در بازه}} \pi$ ۶

$\to x=4k\pi-\pi \to -\pi$

$\boxed{|-\pi-\pi| = 2\pi}$

$\cos^2 x - \sin^2 x\cos 3x = 1 \to \cos^2 x - \sin^2 x\cos 3x = \cos^2 x + \sin^2 x$

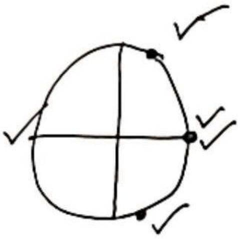
$\to \cos 3x = -1 \to 3x = 2k\pi + \pi \to x = \frac{2}{3}k\pi + \frac{\pi}{3}$

چون $\sin^2 x$ را از طرفین حذف کردیم

باید ریشه‌ی آن را هم در نظر بگیریم. ۷

$\sin^2 x = 0$

$x = k\pi$



۵ جواب

$\frac{1-\tan x}{1+\tan x} = \tan 3x \to \tan(\frac{\pi}{4}-x) = \tan 3x \to 3x = k\pi + \frac{\pi}{4} - x$

$\frac{1-\tan x}{1+\tan x} \downarrow \tan(\frac{\pi}{4}-x)$

$\to 4x = k\pi + \frac{\pi}{4} \to \boxed{x = \frac{k\pi}{4} + \frac{\pi}{16}}$

$\cos x \neq 0 \to x \neq 2k\pi \pm \frac{\pi}{2}$ ۸

$\cos 3x \neq 0 \to 3x \neq 2k\pi \pm \frac{\pi}{2} \to x \neq \frac{k\pi}{3} \pm \frac{\pi}{6}$

$\tan x \neq -1 \to x \neq k\pi + \frac{3\pi}{4}$

$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \to \underbrace{\sin x + \cos x}_{\sqrt{2}\sin(x+\frac{\pi}{4})} = \sqrt{2}\cos 2x$

$\to \cos(\frac{\pi}{4} - x) = \cos 2x \to \cos(x - \frac{\pi}{4}) = \cos 2x$

۳ جواب

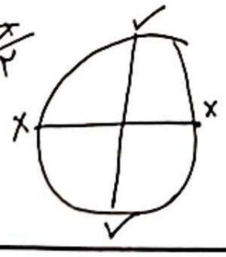
$\to x - \frac{\pi}{4} = 2k\pi + 2x \to x = -2k\pi - \frac{\pi}{4}$ ✗ در بازه نیست ۹

$x - \frac{\pi}{4} = 2k\pi - 2x \to x = \frac{2k\pi}{3} + \frac{\pi}{12} \to \begin{cases} k=0 \to x = \frac{\pi}{12} \\ k=1 \to x = \frac{5\pi}{12} \\ k=2 \to x = \frac{9\pi}{12} \end{cases}$

در این گونه سوالات چهار نقطه‌ی $\frac{k\pi}{2}$ را بررسی می‌کنیم:

الف) $\sin^4 x - \cos^4 x = 1$

$\frac{\pi}{2}$ و $\frac{3\pi}{2}$



ب) $\sin^3 x - \cos^3 x = -1$ ۱۰

$\frac{3\pi}{2}$ و 2π و 0