به نام خدا

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دوازدهم ریاضی ۸

۱-

الف) $1-2\sin^2 x = 3\sin x - 1 \to 0 = 2\sin^2 x + 3\sin x - 2$

$\to (\sin x - 1)(\sin x + 2) = 0 \to \begin{cases} \sin x = -2 & ✗ \\ \sin x = \frac{1}{2} \end{cases} \to \begin{cases} x = 2k\pi + \frac{\pi}{6} \\ x = 2k\pi + \frac{5\pi}{6} \end{cases}$

ب) $2\cos^2 x - 1 + \cos x - 2 = 0 \to 2\cos^2 x + \cos x - 3 = 0$

$\to (\cos x + 3)(\cos x - 2) = 0 \to \begin{cases} \cos x = -\frac{3}{2} & ✗ \\ \cos x = 1 \to x = 2k\pi \end{cases}$

۲-

الف) $-\cos 2x = \sin 4x \to -\cos 2x = 2\sin 2x \cos 2x$

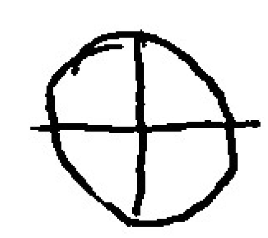
$\cos 2x = 0$ جوابها

$2x = \frac{\pi}{2} + k\pi \to \boxed{x = \frac{\pi}{4} + \frac{k\pi}{2}}$

$-1 = 2\sin 2x \to \sin 2x = -\frac{1}{2} \to -2x = 2k\pi + \frac{\pi}{6}$

$-2x = 2k\pi + \frac{5\pi}{6}$

$\to \boxed{\begin{array}{l} x = k\pi - \frac{\pi}{12} \\ x = k\pi - \frac{5\pi}{12} \end{array}}$



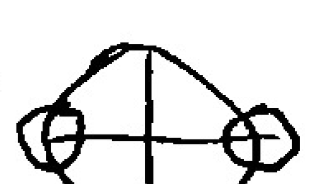
ب) $\sin 2x = \underbrace{\frac{1-2\cos^2 x}{}}_{-\cos 2x}$

$\to 2x = -\frac{\pi}{4} + k\pi$

$\to \boxed{x = -\frac{\pi}{8} + \frac{k\pi}{2}}$

۳-

الف) $\xrightarrow{\sin x \neq 0}$



$\cos x(2\cos x + 1) = 1 \to 2\cos^2 x + \cos x - 1 = 0$

$\begin{cases} \cos x = -1 \to x = \pi + 2k\pi & ✗ \text{ غیرقابل قبول} \\ \cos x = \frac{1}{2} \to \boxed{x = 2k\pi \pm \frac{\pi}{3}} & \text{جواب} \end{cases}$

ب) $\xrightarrow{\div \sin^2 x}$

$2 - \cot x + \cot^2 x = \frac{2}{\sin^2 x} = 2(1 + \cot^2 x)$

$\to \cancel{2} - \cot x + \cancel{\cot^2 x} = 2 + 2\cot^2 x \to 0 = \cot^2 x + \cot x = \cot x(\cot x + 1)$

$\begin{cases} x = -\frac{\pi}{4} + k\pi \\ x = \frac{\pi}{2} + k\pi \end{cases}$

۴-

الف) $\cos(\frac{\pi}{4} + x)\cos(-x + \frac{\pi}{4}) = \frac{1}{4} \to \underbrace{2\cos(\frac{\pi}{4} + x)\sin(\frac{\pi}{4} + x)}_{} = \frac{2}{4}$

$\frac{\pi}{2}$ = مجموع زاویه‌ها

$\sin(2x + \frac{\pi}{2}) = \frac{1}{2} \to \cos 2x = \frac{1}{2}$

$\to 2x = 2k\pi \pm \frac{\pi}{3} \to x = k\pi \pm \frac{\pi}{6}$

ب) $\cos(2x - \frac{\pi}{9}) = \cos(\frac{\pi}{2} + 2x) \to \cancel{2x} - \frac{\pi}{9} = 2k\pi + \frac{\pi}{2} + \cancel{2x}$ غ.ق.ق

$2x - \frac{\pi}{9} = 2k\pi - \frac{\pi}{2} - 2x$ ✓

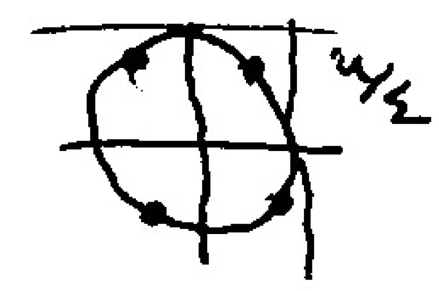
$\to \boxed{x = \frac{k\pi}{2} - \frac{7\pi}{72}}$

$1 = 2\cos 2x + 1$ $\quad D: \sin 2x \neq 0 \to 2x \neq k\pi \to x \neq \frac{k\pi}{2}$ — ۵

$\to \cos 2x = 0 \to 2x = \frac{\pi}{2} + k\pi \to \boxed{x = \frac{\pi}{4} + \frac{k\pi}{2}}$

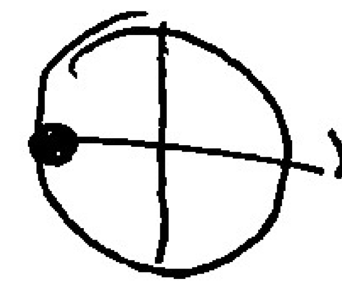
مجموعه جواب $\{x \mid x = \frac{\pi}{4} + \frac{k\pi}{2}, k \in Z\}$

$\tan \frac{x}{4} = \dfrac{\cos \frac{x}{2} + 1}{\underbrace{\sin \frac{x}{2}}_{\cot x/4}} \to \tan x/4 = \cot x/4$ — ۶



$\frac{x}{4} = \frac{\pi}{4} + \frac{k\pi}{2}$

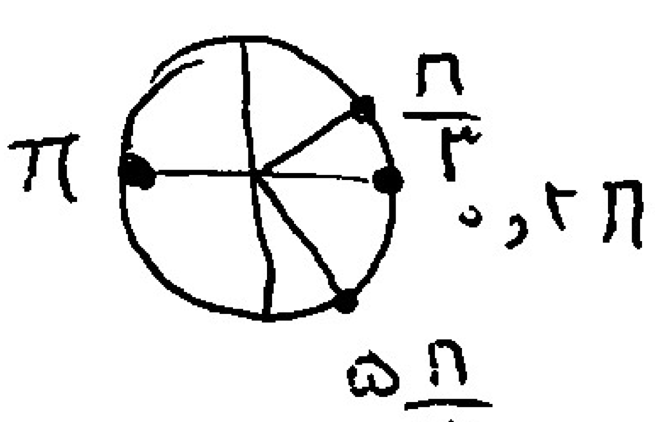
$\to x = \pi + 2k\pi$



$x = -\pi, \pi \to |x_1 - x_2| = \boxed{2\pi}$

$\sin^2 x = -\sin^2 x \cos 3x \xrightarrow{\sin x = 0} 1 = -\cos 3x$ — ۷

$\begin{cases} x = k\pi \\ 3x = \pi + 2k\pi \to x = \frac{\pi}{3} + \frac{2k\pi}{3} \end{cases}$



$S = 5\pi$

$+\tan(-x + \frac{\pi}{4}) = \tan 3x$ — ۸

$3x = k\pi - x + \frac{\pi}{4} \to 4x = k\pi + \frac{\pi}{4} \to x = \frac{k\pi}{4} + \frac{\pi}{16}$

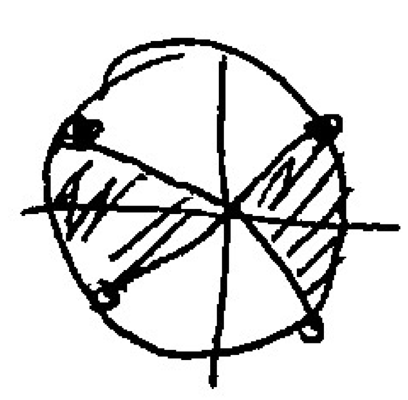
— ۹

$\xrightarrow{\cos 2x \geq 0} 1 + \sin 4x = 2\cos^2 2x \to \sin 4x = \cos 4x$

$\cos(\frac{\pi}{2} - 4x)$

$\frac{\pi}{2} - 4x = 2k\pi + 4x \to x = \frac{\pi}{16} + \frac{k\pi}{4}$

$\frac{\pi}{2} - 4x = 2k\pi - 4x \to x = k\pi - \frac{\pi}{4}$ ✓ $\quad \frac{\pi}{4} + k\pi \neq x$



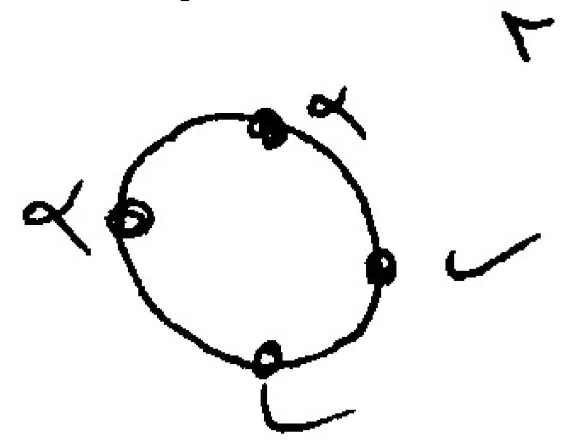
$D:$

$x = \frac{3\pi}{4}, \frac{\pi}{16}$

الف) $\cos 2x = -1 \to 2x = \pi + 2k\pi \to x = \frac{\pi}{2} + k\pi$ — ۱۰

$\xrightarrow{0 \leq x \leq 2\pi} x = \frac{\pi}{2}, \frac{3\pi}{2}$

ب)



$x = 0, \frac{3\pi}{2}, 2\pi$