نام و نام خانوادگی: حسام رفیع زاده «به نام خدا» مقطع: دوازدهم پسر A

شماره: ۱۲

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الف) $\cos 2x = 3\sin x - 1 \rightarrow 1 - 2\sin^2 x = 3\sin x - 1 \rightarrow 2\sin^2 x + 3\sin x - 2 = 0$

$\xrightarrow{\text{روش}} \sin^2 x + 2\sin x - 4 = 0 \rightarrow (\sin x + 4)(\sin x - 1) = 0 \rightarrow \sin x = \frac{-4}{2} = -2$ غ ق ق

$\sin x = \frac{1}{2}$

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$\rightarrow x = 2k\pi + \frac{\pi}{6} \;,\; x = 2k\pi + \pi - \frac{\pi}{6}$

ب) $\cos 2x + \cos x - 2 = 0 \rightarrow 2\cos^2 x + \cos x - 3 = 0 \xrightarrow{\text{روش}} (\cos x + 3)(\cos x - 2) = 0$

$\rightarrow \cos x = \frac{-3}{2} =$ غ ق ق

$\cos x = \frac{2}{2} = 1 \rightarrow x = 2k\pi$

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الف) $\sin^4 x - \cos^4 x = \sin 4x \rightarrow (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) = \sin 4x$

$\rightarrow -\cos 2x = \sin 4x \rightarrow \sin\left(\frac{3\pi}{2} + 2x\right) = \sin 4x \rightarrow 4x = 2k\pi + \frac{3\pi}{2} + 2x$

$4x = 2k\pi + \pi - \frac{3\pi}{2} - 2x$

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$\rightarrow 2x = 2k\pi + \frac{3\pi}{2} \rightarrow x = k\pi + \frac{3\pi}{4}$

$\rightarrow 6x = 2k\pi - \frac{\pi}{2} \rightarrow x = \frac{k\pi}{3} - \frac{\pi}{12}$

$\cos 2x(2\sin 2x + 1) = 0$

$\cos 2x = 0 \rightarrow x = \frac{k\pi}{2} + \frac{\pi}{4}$

$\sin 2x = \frac{-1}{2} \rightarrow 2k\pi + \frac{7\pi}{6} \,,\, 2k\pi - \frac{\pi}{6}$

ب) $2\cos^2 x + 2\sin x \cdot \cos x = 1 \rightarrow -2\sin x \cdot \cos x = 2\cos^2 x - 1$

$\rightarrow -\sin 2x = \cos 2x \rightarrow \sin 2x = \sin\left(\frac{3\pi}{2} + 2x\right) \rightarrow 2x = 2k\pi + \frac{3\pi}{2} + 2x \;\varnothing$

$\rightarrow 2x = 2k\pi + \pi - \frac{3\pi}{2} - 2x$

$\rightarrow 4x = 2k\pi + \pi - \frac{3\pi}{2}$

$\rightarrow x = \frac{k\pi}{2} - \frac{\pi}{8}$

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الف) $\cot x\,(2\cos x+1)=\frac{1}{\sin x} \to \frac{\cos x}{\sin x}(2\cos x+1)=\frac{1}{\sin x} \xrightarrow[]{\overset{①}{\sin x\neq 0}}$

$2\cos^2 x+\cos x-1=0 \to \cos x=-1 \to \varnothing$ چون $\sin x=0$ ①

$\to \cos x=\frac{1}{2} \to \boxed{x=2k\pi\pm\frac{\pi}{3}}$

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ب) $2\sin^2 x-\sin x\cdot\cos x+\cos^2 x=2 \to \sin^2 x+1-\sin x\cdot\cos x=2$

$\to \sin^2 x-\sin x\cdot\cos x=1 \xrightarrow{\div\cos^2 x} \cancel{\tan^2 x}-\tan x=1+\cancel{\tan^2 x} \to \tan x=-1 \to \boxed{x=k\pi-\frac{\pi}{4}}$

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الف) $\cos(\frac{\pi}{4}+x)\cos(x-\frac{\pi}{4})=\frac{1}{4} \to (\frac{\sqrt{2}}{2}\cos x-\frac{\sqrt{2}}{2}\sin x)(\frac{\sqrt{2}}{2}\cos x+\frac{\sqrt{2}}{2}\sin x)=\frac{1}{4}$

$\to \frac{1}{2}\cos^2 x-\frac{1}{2}\sin^2 x=\frac{1}{4} \to \cos 2x=\frac{1}{2} \to 2x=2k\pi\pm\frac{\pi}{3} \to \boxed{x=k\pi\pm\frac{\pi}{6}}$

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ب) $\cos(2x-\frac{\pi}{9})=-\sin 2x \to \cos(2x-\frac{\pi}{9})=\cos(\frac{\pi}{2}+2x)$

$\to 2x-\frac{\pi}{9}=2k\pi\pm(2x+\frac{\pi}{2}) \to 2x=2k\pi-2x-\frac{\pi}{2} \to \boxed{x=\frac{k\pi}{2}-\frac{\pi}{8}}$

$\hookrightarrow x=\frac{k\pi}{2}-\frac{7\pi}{12}$

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$\frac{\sin 4x+\sin 2x}{\sin 2x}-\sin^2 x=\cos^2 x \to \sin 4x+\sin 2x=\sin 2x \to \sin 4x=0 \to$

$x=2k\pi\ ,\ 2k\pi+\pi \xrightarrow[\text{چون}]{\text{اما}} \boxed{x=\varnothing}$

مخرج صفر شود

$\cos 2x=0 \to x=\frac{k\pi}{2}+\frac{\pi}{4}$

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$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}=\frac{1}{\sin\frac{x}{2}}+\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} \to \sin^2\frac{x}{2}=(1+\cos\frac{x}{2})^2 \to \sin^2\frac{x}{2}=1+\cos^2\frac{x}{2}+2\cos\frac{x}{2}$

$\to \cancel{1}-\cos^2\frac{x}{2}=\cancel{1}+\cos^2\frac{x}{2}+2\cos\frac{x}{2} \to 2\cos^2\frac{x}{2}+2\cos\frac{x}{2}=0 \to \underbrace{\cos\frac{x}{2}}_{\to 0}(\underbrace{\cos\frac{x}{2}+1}_{\to -1})=0$

غ ق ق مخرج صفر

$\cos\frac{x}{2}=0 \to \frac{x}{2}=k\pi+\frac{\pi}{2} \to x=2k\pi+\pi$

$\xrightarrow{k=0} x=\pi\ ,\ x=-\pi \to |\pi-(-\pi)|=\boxed{2\pi}$

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۷- $\cos^2 x - \sin^2 x \cos^3 x = 1 \to -\sin^2 x \cdot \cos^3 x = \underbrace{1-\cos^2 x}_{\sin^2 x} \xrightarrow[\text{غیرجواب}]{\sin x = 0} \cos^3 x = 0$

$\to \cos x = 0 \quad \to x = \frac{k\pi}{4} \quad \to$ $\boxed{\text{تعداد جواب ها} = 5}$

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۸- $\frac{1-\tan x}{1+\tan x} = \tan 3x \to \tan(\frac{\pi}{4} - x) = \tan 3x \to 3x = k\pi + \frac{\pi}{4} - x$

$\to 4x = k\pi + \frac{\pi}{4} \to x = \frac{k\pi}{4} + \frac{\pi}{16}$

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۹- $\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \xrightarrow{()^2} 1+\sin 2x = 2\cos^2 2x \to \sin 2x = 2\cos^2 2x - 1$

$\to \sin 2x = \cos 4x \to \sin 2x = \sin(\frac{\pi}{2} + 4x) \to 2x = 2k\pi + \frac{\pi}{2} + 4x$

$\to 2x = 2k\pi + \pi - \frac{\pi}{2} - 4x$

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$\to x = -k\pi - \frac{\pi}{4}$

$\to x = \frac{k\pi}{3} + \frac{\pi}{12}$ $\quad \to \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12} \xrightarrow{(0,\pi)}$ ۲ جواب ~~۳ جواب~~

$\sin 2x = -1 \to x = k\pi - \frac{\pi}{4} \to \frac{3\pi}{4}$

$x = k\pi + \frac{\pi}{12} \to \frac{\pi}{12}$

۱۰- الف) $\sin^4 x - \cos^4 x = 1 \to \underbrace{(\sin^2 x + \cos^2 x)}_{1}(\sin^2 x - \cos^2 x) = 1 \to \cos 2x = -1$

$2x = 2k\pi + \pi \to x = k\pi + \frac{\pi}{2} \to \{\frac{\pi}{2}, \frac{3\pi}{2}\}$

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ب) $\sin^3 x - \cos^3 x = -1 \to \sin^3 x - \cos^3 x + \sin^2 x + \cos^2 x = 0 \to$

$x = 2\pi, 0, \frac{3\pi}{2}$

$\sin^2 x(\sin x + 1) - \cos^2 x(\cos x - 1) = 0 \to \sin^2 x(\sin x + 1) = \cos^2 x(\cos x - 1)$

$\to \frac{\sin^2 x}{\cos^2 x} = \frac{\cos x - 1}{\sin x + 1} \to \frac{\overbrace{\sin^2 x}^{\text{مثبت}}}{\underbrace{\cos^2 x}_{\text{مثبت}}} = \frac{-2\sin^2\frac{x}{2}}{\underbrace{(\sin\frac{x}{2} + \cos\frac{x}{2})^2}_{\text{مثبت}}} \to -2\sin^2\frac{x}{2}$ باید مثبت باشد

$\to -2\underbrace{\sin^2\frac{x}{2}}_{\text{مثبت}} > 0 \to$ غیرممکن $\to \boxed{x = \varnothing}$