نام و نام خانوادگی ......آترین جلیلی..... پاسخنامه تشریحی تکلیف شماره ...۱۷.. کلاس .دوازدهم تجربی ۸.........

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$$8\cos x-\tan^2 x=1 \rightarrow 8\cos x-\frac{\overbrace{1-\cos^2 x}^{\sin^2 x}}{\cos^2 x}=1 \rightarrow \frac{8\cos^3 x-1+\cos^2 x}{\cos^2 x}=1$$

$$\rightarrow 8\cos^3 x-1+\cos^2 x=\cos^2 x \rightarrow 8\cos^3 x=1 \rightarrow \cos x=\frac{1}{2} \rightarrow x=2k\pi\pm\frac{\pi}{3}$$

$$\rightarrow \begin{cases} x=2k\pi+\frac{\pi}{3} \xrightarrow[\text{بین } [0,2\pi]]{k=0} x=\frac{\pi}{3} \\ x=2k\pi-\frac{\pi}{3} \xrightarrow[\text{بین } [0,2\pi]]{k=1} x=\frac{5\pi}{3} \end{cases}$$

دو جواب دارد

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$$5\sin^2 x+2\cos 3x=-2 \xrightarrow{(I),(II)} 5\frac{(1-\cos 2x)}{2}+8\cos^3 x-6\cos x=-2 \xrightarrow{\times 2} 5-5\cos 2x+16\cos^3 x-12\cos x+4=0$$

$\cos 3x=4\cos^3 x-3\cos x \ (I)$

$\sin^2 x=\frac{1-\cos 2x}{2} \ (II)$

$$\rightarrow 16\cos^3 x-5\underbrace{\cos 2x}_{2\cos^2 x-1}-12\cos x+9=0$$

$$\rightarrow 16\cos^3 x-10\cos^2 x-12\cos x+14=0 \xrightarrow{\div 2} 8\cos^3 x-5\cos^2 x-6\cos x+7=0$$

$$\xrightarrow[\cos x=-1]{} (\cos x+1)(\underbrace{8\cos^2 x-13\cos x+7}_{\Delta<0}) \rightarrow \cos x=-1 \Rightarrow x=2k\pi+\pi \xrightarrow[{[-\pi,\pi]}]{k=-1,0} x=\pi, -\pi$$

دو جواب دارد

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$$2\sin x\cos 2x+\sin x=1 \rightarrow \sin x(2\cos 2x+1)=1 \xrightarrow{(I)} \sin x(3-4\sin^2 x)=1$$

$\cos 2x=1-2\sin^2 x \rightarrow 2\cos 2x+1=3-4\sin^2 x \ (I)$ $\quad \rightarrow 4\sin^3 x-3\sin x+1=0$

$$\rightarrow \begin{cases} \sin x=-1 \rightarrow x=2k\pi+\frac{3\pi}{2} \xrightarrow[k=0]{[0,2\pi]} x=\frac{3\pi}{2}=\frac{9\pi}{6} \\ \sin x=\frac{1}{2} \rightarrow \begin{cases} x=2k\pi+\frac{\pi}{6} \xrightarrow{k=0} x=\frac{\pi}{6} \\ x=2k\pi+\frac{5\pi}{6} \xrightarrow{k=0} x=\frac{5\pi}{6} \end{cases} \end{cases}$$

$$\Rightarrow \sum x=\frac{9\pi}{6}+\frac{\pi}{6}+\frac{5\pi}{6}=\frac{15\pi}{6}=\frac{5\pi}{2}$$

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$$\frac{1}{\sin(2x+\frac{\pi}{2})}+\frac{1}{\cos(4x+\frac{\pi}{2})}=\frac{1}{\cos 2x}+\frac{1}{-\sin 4x}=0 \rightarrow \frac{1}{\cos 2x}=\frac{1}{\sin 4x} \rightarrow \sin 4x=\cos 2x$$

$$\rightarrow 2\sin 2x\cos 2x=\cos 2x \xrightarrow{\cos 2x\neq 0} 2\sin 2x=1 \rightarrow \sin 2x=\frac{1}{2}$$

$$\rightarrow \begin{cases} 2x=2k\pi+\frac{\pi}{6} \xrightarrow{k=0} 2x=\frac{\pi}{6} \rightarrow x=\frac{\pi}{12} \Rightarrow \alpha=\frac{5\pi}{12}-\frac{\pi}{12}=\frac{\pi}{3} \rightarrow 2\alpha=\frac{2\pi}{3} \\ 2x=2k\pi+\frac{5\pi}{6} \xrightarrow{k=0} 2x=\frac{5\pi}{6} \rightarrow x=\frac{5\pi}{12} \end{cases}$$

$$\rightarrow \tan 2\alpha=\tan\frac{2\pi}{3}=-\sqrt{3}$$

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$$\cos(x+\frac{\pi}{4})=\frac{1}{\sqrt{3}} \rightarrow \cos x\cos\frac{\pi}{4}-\sin x\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}(\cos x-\sin x)=\frac{1}{\sqrt{3}} \rightarrow \cos x-\sin x=\frac{\sqrt{6}}{3} \ (I)$$

$$\rightarrow m(\cos x-\sin x)-3\sqrt{6}\sin 2x=\sqrt{6} \xrightarrow{I,II} m\left(\frac{\sqrt{6}}{3}\right)-\sqrt{6}=3\sqrt{6}\left(\frac{1}{3}\right) \rightarrow \boxed{m=6}$$

$$(\cos x-\sin x)^2=\left(\frac{\sqrt{6}}{3}\right)^2 \rightarrow \underbrace{\cos^2 x+\sin^2 x}_{1}-2\sin x\cos x=\frac{2}{3} \rightarrow \sin 2x=\frac{1}{3} \ (II)$$

$$\sin\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{3}\right)=1 \qquad \sin A\cos B=\frac{1}{2}\left[\sin(A+B)+\sin(A-B)\right]$$

$$=\frac{1}{2}\left[\sin\left(2x-\frac{\pi}{6}\right)+\sin\left(\frac{\pi}{2}\right)\right]=1$$

$$\xrightarrow{\times 2} \sin\left(2x-\frac{\pi}{6}\right)+1=2 \to \sin\left(2x-\frac{\pi}{6}\right)=1 \to 2k\pi+\frac{\pi}{2}=2x-\frac{\pi}{6}$$

$$\to 2x=2k\pi+\frac{\pi}{2}+\frac{\pi}{6}=2k\pi+\frac{2\pi}{3} \to x=k\pi+\frac{\pi}{3} \begin{cases} \to k=0 \to x=\frac{\pi}{3} \\ \to k=1 \to x=\frac{4\pi}{3} \end{cases}$$

دو جواب دارد

$$1\sin x+\sqrt{3}\cos x=\sqrt{3} \longrightarrow \frac{b}{a}=\sqrt{3} \to \alpha=\tan^{-1}(\sqrt{3})=\frac{\pi}{3}$$

$$a\sin x+b\cos x=\frac{a}{|a|}\sqrt{a^2+b^2}\,\sin(x+\alpha),\ \tan\alpha=\frac{b}{a}$$

$$\to \sin x+\sqrt{3}\cos x=\sqrt{1+3}\,\sin\left(x+\frac{\pi}{3}\right)=\sqrt{3} \to \sin\left(x+\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$$

$$\to \begin{cases} x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4} \to x=2k\pi-\frac{\pi}{12} \xrightarrow[{[-\pi,2\pi]}]{k=0,1} x=\frac{-\pi}{12},\ \frac{23\pi}{12} \\ x+\frac{\pi}{3}=2k\pi+\frac{3\pi}{4} \to x=2k\pi+\frac{5\pi}{12} \xrightarrow[k=0]{} x=\frac{5\pi}{12} \end{cases} \Sigma x=\frac{27\pi}{12}=\frac{9\pi}{4}$$

$$4\sin x\,\underbrace{\sin\left(\frac{3\pi}{2}-x\right)}_{-\cos x}=1 \to -4\sin x\cos x=1 \to 2\sin 2x=-1 \to \sin 2x=\frac{-1}{2}$$

$$\to \begin{cases} 2x=2k\pi+\frac{7\pi}{6} \to x=k\pi+\frac{7\pi}{12} \xrightarrow[k=0,1]{[0,2\pi]} \begin{cases} x=\frac{7\pi}{12} \\ x=\frac{19\pi}{12} \end{cases} \\ 2x=2k\pi+\frac{11\pi}{6} \to x=k\pi+\frac{11\pi}{12} \xrightarrow[k=0,1]{} \begin{cases} x=\frac{11\pi}{12} \\ x=\frac{23\pi}{12} \end{cases} \end{cases} \Rightarrow \Sigma x=\frac{60\pi}{12}=5\pi$$

$$(\underbrace{1+\cos 2\alpha}_{2\cos^2\alpha})(\underbrace{1+\cos 4\alpha}_{2\cos^2 2\alpha})(\underbrace{1+\cos 8\alpha}_{2\cos^2 4\alpha})=\frac{1}{8} \Rightarrow \cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha=\frac{1}{64}$$

$$\to \cos\alpha\cos 2\alpha\cos 4\alpha=\pm\frac{1}{8}$$

$$\cos\alpha=\frac{1}{2} \to \frac{1}{2}\times\left(\frac{-1}{2}\right)\left(\frac{-1}{2}\right)=\frac{1}{8}\ \checkmark \to \cos\alpha=\frac{1}{2} \to \alpha=\frac{\pi}{3}$$

$$\cos 2\alpha=2\cos^2\alpha-1=\frac{-1}{2}$$

$$\cos 4\alpha=2\cos^2 2\alpha-1=\frac{-1}{2}$$

$$\begin{cases} \cos\alpha=-\frac{1}{2} \to \alpha=\frac{2\pi}{3} \\ \cos 2\alpha=\cos\frac{4\pi}{3}=-\frac{1}{2} \\ \cos 4\alpha=\cos\frac{8\pi}{3}=-\frac{1}{2} \end{cases} \Rightarrow \left(\frac{-1}{2}\right)\left(\frac{-1}{2}\right)\left(\frac{-1}{2}\right)=\frac{-1}{8}\ \checkmark$$

پس مجموعه جواب ها در $[0,\pi]$

$$A=\left\{\frac{\pi}{3},\frac{2\pi}{3}\right\}$$

$$A_{max}=\frac{2\pi}{3}$$

$$\tan 3x\tan x=1 \longrightarrow \tan 3x=\frac{1}{\tan x}=\cot x=\tan\left(\frac{\pi}{2}-x\right) \to \tan 3x=\tan\left(\frac{\pi}{2}-x\right)$$

$$\to 3x=k\pi+\frac{\pi}{2}-x \to 4x=k\pi+\frac{\pi}{2} \to x=\frac{k\pi}{4}+\frac{\pi}{8} \xrightarrow{[\pi,2\pi]} k=4,5,6,7$$

$$\to \begin{cases} x=\pi+\frac{\pi}{8} \\ x=\frac{5\pi}{4}+\frac{\pi}{8} \\ x=\frac{6\pi}{4}+\frac{\pi}{8} \\ x=\frac{7\pi}{4}+\frac{\pi}{8} \end{cases} \Rightarrow \Sigma x=\frac{\pi}{2}+\frac{22\pi}{4}=\frac{24\pi}{4}=6\pi$$